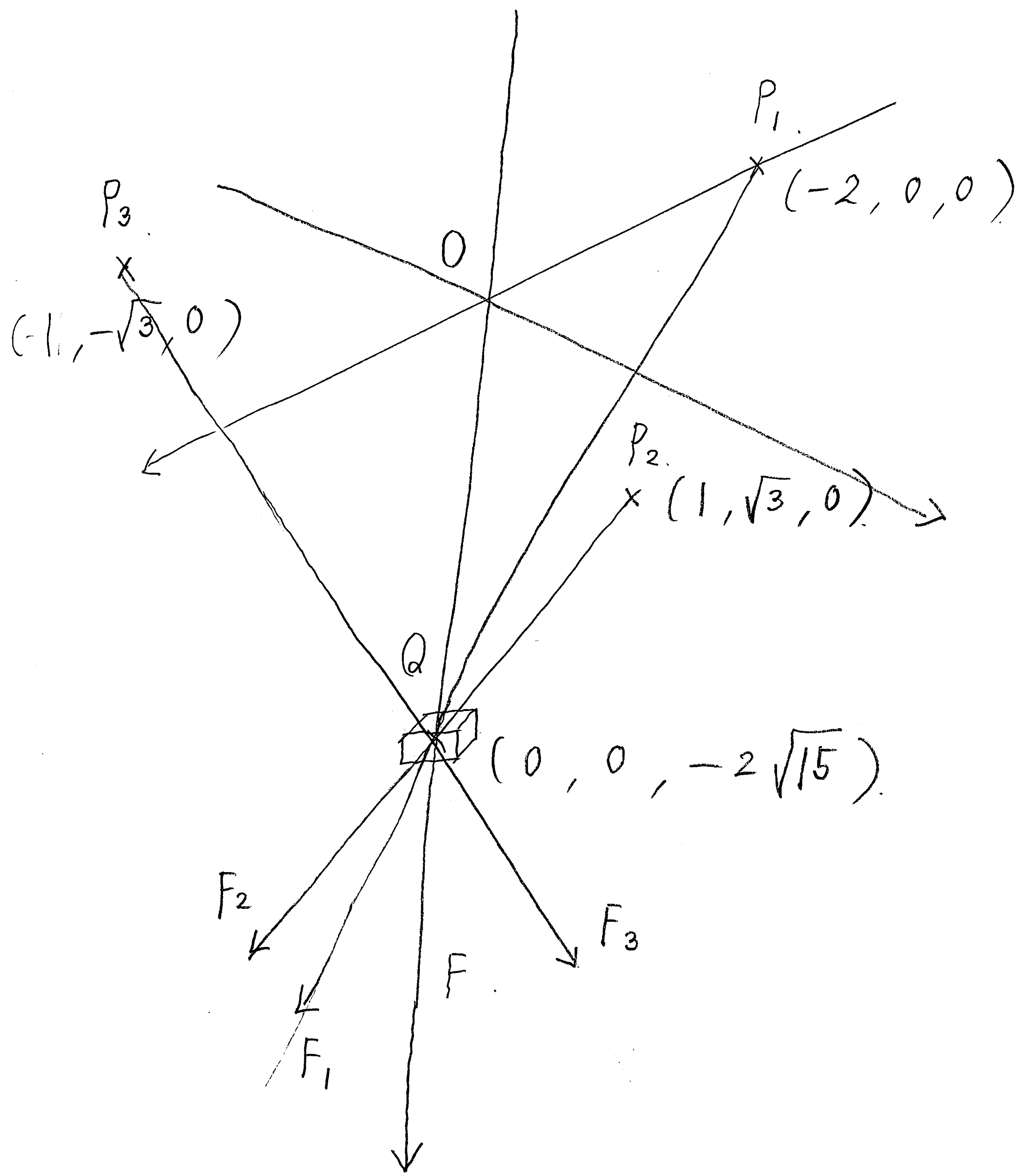
Explanation of

Problem # 9

Lesson 2 , Part II



$\vec{F}$: the force in the direction of $\overrightarrow{OQ}$
due to the load

$$|\vec{F}| = 300 \text{ lb.}$$

$\vec{F_1}$: the force in the direction of $\overrightarrow{P_1Q}$

(the vector describing the force
on the cable anchored at P_1)

$\vec{F_2}$: " " $\overrightarrow{P_2Q}$

(" "
" " " at P_2)

$\vec{F_3}$: " " $\overrightarrow{P_3Q}$

(" "
" " at P_3)

Now we have

$$\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3}.$$

i.e.

$$(\star)\quad |\vec{F}| \frac{\vec{F}}{|\vec{F}|} = |\vec{F_1}| \frac{\vec{F_1}}{|\vec{F_1}|} + |\vec{F_2}| \frac{\vec{F_2}}{|\vec{F_2}|} + |\vec{F_3}| \frac{\vec{F_3}}{|\vec{F_3}|}$$

This last equation (☆) looks useless, since we don't know what

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$$\vec{F_1}\ ,\ \vec{F_2}\ ,\ \vec{F_3}.$$

We set

$$\vec{a_1} = \overrightarrow{P_1Q} = \langle 2, 0, -2\sqrt{15} \rangle$$

$$\vec{a_2} = \overrightarrow{P_2Q} = \langle -1, -\sqrt{3}, -2\sqrt{15} \rangle$$

$$\vec{a_3} = \overrightarrow{P_3Q} = \langle -1, \sqrt{3}, -2\sqrt{15} \rangle$$

Then we realize (this is the key ☺)

$$\frac{\vec{F_1}}{|\vec{F_1}|} = \frac{\vec{a_1}}{|\vec{a_1}|} = \left\langle \frac{1}{4}, 0, -\frac{\sqrt{15}}{4} \right\rangle$$

while $|\vec{a_1}| = 8$

$$\frac{\vec{F_2}}{|\vec{F_2}|} = \frac{\vec{a_2}}{|\vec{a_2}|} = \left\langle -\frac{1}{8}, -\frac{\sqrt{3}}{8}, -\frac{\sqrt{15}}{4} \right\rangle$$

while $|\vec{a_2}| = 8$

$$\frac{\vec{F_3}}{|\vec{F_3}|} = \frac{\vec{a_3}}{|\vec{a_3}|} = \left\langle -\frac{1}{8}, \frac{\sqrt{3}}{8}, -\frac{\sqrt{15}}{4} \right\rangle$$

while $|\vec{a_3}| = 8$

Now we have

$$|\vec{F}| \cdot \frac{\vec{F}}{|\vec{F}|} = 300 \cdot \langle 0, 0, -1 \rangle$$

$$|\vec{F_1}| \frac{\vec{F}}{|\vec{F_1}|} = |\vec{F_1}| \frac{\vec{a_1}}{|\vec{a_1}|}$$

$$= s \left\langle \frac{1}{4}, 0, -\frac{\sqrt{15}}{4} \right\rangle$$

$$|\vec{F_2}| \cdot \frac{\vec{F_2}}{|\vec{F_2}|} = |\vec{F_2}| \frac{\vec{a_2}}{|\vec{a_2}|}$$

$$= t \left\langle -\frac{1}{8}, -\frac{\sqrt{3}}{8}, -\frac{\sqrt{15}}{4} \right\rangle$$

$$|\vec{F_3}| \frac{\vec{F_3}}{|\vec{F_3}|} = |\vec{F_3}| \frac{\vec{a_3}}{|\vec{a_3}|}$$

$$= u \left\langle -\frac{1}{8}, \frac{\sqrt{3}}{8}, -\frac{\sqrt{15}}{4} \right\rangle$$

where we set

$$\begin{cases} |\vec{F_1}| = s \\ |\vec{F_2}| = t \\ |\vec{F_3}| = u \end{cases} \quad \text{(unknowns)}$$

Therefore, the equation (☆) leads to 5

$$\begin{cases} s\left(\frac{1}{4}\right) + t\left(-\frac{1}{8}\right) + u\left(-\frac{1}{8}\right) = 0 \\ s \cdot 0 + t\left(-\frac{\sqrt{3}}{8}\right) + u\left(-\frac{\sqrt{15}}{8}\right) = 0 \\ s\left(-\frac{\sqrt{15}}{4}\right) + t\left(-\frac{\sqrt{15}}{4}\right) + u\left(-\frac{\sqrt{15}}{4}\right) \\ \qquad = -300 \end{cases}$$

$\rightarrow$

$$\begin{cases} 2s - t - u = 0 \\ \quad - t + u = 0 \\ s + t + u = -300 \cdot \left(-\frac{4}{\sqrt{15}}\right) \\ \qquad = 80\sqrt{15} \end{cases}$$

$\rightarrow$

$$s = t = u = \frac{80\sqrt{15}}{3}$$

That is to say,

$$|\vec{F_1}| = |\vec{F_2}| = |\vec{F_3}| = \frac{80\sqrt{15}}{3}$$

Finally we conclude

$$\vec{F_1} = |\vec{F_1}| \frac{\vec{F_1}}{|\vec{F_1}|}$$

$$= \frac{80\sqrt{15}}{3} \left\langle \frac{1}{4}, 0, -\frac{\sqrt{15}}{4} \right\rangle$$

$$= \left\langle \frac{20\sqrt{15}}{3}, 0, -100 \right\rangle$$

$$\vec{F_2} = |\vec{F_2}| \frac{\vec{F_2}}{|\vec{F_2}|}$$

$$= \frac{80\sqrt{15}}{3} \left\langle -\frac{1}{8}, -\frac{\sqrt{3}}{8}, -\frac{\sqrt{15}}{8} \right\rangle$$

$$= \left\langle -\frac{10\sqrt{15}}{3}, -10\sqrt{5}, -100 \right\rangle$$

$$\vec{F_3} = \frac{80\sqrt{15}}{3} \left\langle -\frac{1}{8}, \frac{\sqrt{3}}{8}, -\frac{\sqrt{15}}{8} \right\rangle$$

$$= \left\langle -\frac{10\sqrt{15}}{3}, 10\sqrt{5}, -100 \right\rangle$$

$\square$