MA161 **EXAM 2** **October 15, 1997**

Name: SOLUTION KEY

ID #: ____________________

Recitation Instructor ________________ Time of Recitation ________________

Section #: ____________________

Instructions:

1. Fill in your name, student ID number and division and section number on the mark–sense sheet. Also fill out the information requested above.
2. This booklet consists of 6 pages. There are 14 questions, each worth 7 points.
3. Mark your answers on the mark–sense sheet. Please show your working in this booklet.
4. No books, notes or calculator may be used.
5. When you are finished with the exam hand this booklet and the mark–sense sheet, in person, to your instructor.

1. If $f(x) = x^3 \ln x$, $f'(x) =$

$$f'(x) = (3x^2)(\ln x) + (x^3)\left(\frac{1}{x}\right)$$
$$= 3x^2 \ln x + x^2$$

A. $3x$
B. $3x^2 \ln x + x^2$
C. $x^2 \ln x + 3x^2$
D. $\frac{3}{x^2}$
E. $3x^2 + \frac{1}{x}$

2. If $f(t) = \frac{2+t}{3-t}$, $f'(t) =$

$$f'(t) = \frac{(1)(3-t) - (2+t)(-1)}{(3-t)^2}$$
$$= \frac{3-t+2+t}{(3-t)^2}$$
$$= \frac{5}{(3-t)^2}$$

A. $\frac{5}{(3-t)^2}$
B. $\frac{1-2t}{(3-t)^2}$
C. $\frac{5+2t}{3-t}$
D. $\frac{-1}{(3-t)}$
E. $\frac{5+2t}{(3-t)^2}$

3. If $f(x) = \ln(\cos(2x))$, $f'(x) =$

$$f'(x) = \frac{1}{\cos 2x}(-\sin 2x)(2)$$
$$= -2\tan 2x$$

A. $\tan(2x)$
B. $-2\tan(2x)$
C. $2\tan(2x)$
D. $\frac{-\sin 2x}{2x}$
E. $\frac{1}{\cos(2x)}$

4. Given that $f'(8) = a$ and $f'(2) = b$, evaluate $\frac{d}{dx}(f(x^3))$ at $x = 2$.

$\frac{d}{dx}\left(f(x^3)\right) = f'(x^3) \cdot 3x^2$

$\frac{d}{dx}\left(f(x^3)\right)\Big|_{x=2} = f'(8) \cdot 12 = 12a$

A. $12a$
B. $3b$
C. ab
D. $12b$
E. $3a$

5. If $g(x) = \sin(x^4)$ then $g''(x) =$

$g'(x) = (\cos x^4)(4x^3)$

$g''(x) = (-\sin x^4)(4x^3)(4x^3) + (\cos x^4)(12x^2)$

$= -16x^6 \sin x^4 + 12x^2 \cos x^4$

A. $4x^3 \cos x^4 + \cos(x^4)$
B. $12x^2 \cos(x^4) - 16x^6 \sin(x^4)$
C. $4x^2 \sin(x^4) + \cos(x^4)$
D. $\sin(4x^3) + x^4 \cos(x^4)$
E. None of the above

6. An object is moving along the x-axis. At time t its position is given by

$$h(t) = -1/t^2 + 3t + 8.$$

Its acceleration at time t is

velocity $= h'(t) = \frac{2}{t^3} + 3$

acceleration $= h''(t) = \frac{-6}{t^4}$

A. $-\frac{3}{t^4}$
B. $-1/t^3$
C. 3
D. $-6/t^4$
E. $-\frac{2}{t^3}$

7. If $x^2 + y^2x + 3y^2 = 5$, then $\dfrac{dy}{dx} =$

$\Rightarrow 2x + 2y\dfrac{dy}{dx}x + y^2 + 6y\dfrac{dy}{dx} = 0$

$\rightarrow \dfrac{dy}{dx}(2xy + 6y) = -2x - y^2$

$\rightarrow \dfrac{dy}{dx} = \dfrac{-2x - y^2}{2xy + 6y}$

A. $\dfrac{-y^2 - 2x}{2xy + 6y}$

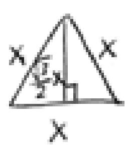
B. $\dfrac{-2x}{2x + 6}$

C. $\dfrac{y^2 + 2x}{2xy + 6y}$

D. $\dfrac{5 - x^2}{3 + x}$

E. $\dfrac{-2x + y^2}{2xy + 6y}$

8. The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. At what rate is the area of the triangle increasing when the length of the sides is 8 cm?



know: $\dfrac{dx}{dt} = 2$ want: $\dfrac{dA}{dt}$ at $x = 8$

$A = \left(\frac{1}{2}x\right)\left(\frac{\sqrt{3}}{2}x\right) = \dfrac{\sqrt{3}}{4}x^2$

$\dfrac{dA}{dt} = \dfrac{\sqrt{3}}{2}x\dfrac{dx}{dt} = \dfrac{\sqrt{3}}{2}(8)(2) = 8\sqrt{3}$

A. $1/4$ cm^2/sec

B. $2\sqrt{3}$ cm^2/sec

C. $8\sqrt{3}$ cm^2/sec

D. 16 cm^2/sec

E. 5 cm^2/sec

9. Gas is being pumped into a spherical balloon at the rate of 8 ft^3/min. How fast is the radius of the balloon increasing when the radius of the balloon is 2 ft?

know: $\dfrac{dV}{dt} = 8$ want: $\dfrac{dr}{dt}$ at $r = 2$

$V = \frac{4}{3}\pi r^3 \longrightarrow \dfrac{dV}{dt} = 4\pi r^2\dfrac{dr}{dt}$

$\longrightarrow 8 = 4\pi(2)^2\dfrac{dr}{dt}$

$\longrightarrow \dfrac{dr}{dt} = \dfrac{8}{4\pi(2)^2} = \dfrac{1}{2\pi}$

A. $\dfrac{1}{2\pi}$ ft/min

B. $\dfrac{8}{\pi}$ ft/min

C. $-\dfrac{1}{2\pi}$ ft/min

D. $\dfrac{\pi}{8}$ ft/min

E. $4\pi^2$ ft/min

10. Given that $(27)^{1/3} = 3$, use linear approximation to approximate $(25)^{1/3}$.

Let $f(x) = x^{1/3}$. Note: $(25)^{1/3} = (27-2)^{1/3}$

$f'(x) = \frac{1}{3}x^{-2/3}$

$(27-2)^{1/3} \approx f(27) + f'(27)\,dx$

$= 3 + \left(\frac{1}{3}27^{-2/3}\right)(-2)$

$= 3 - \frac{2}{27}$

A. $3 - \frac{1}{27}$

B. $3 - \frac{5}{27}$

C. $3 - \frac{2}{27}$

D. $3 - \frac{6}{27}$

E. $3 - \frac{4}{27}$

11. The critical numbers of the function xe^{3x} are

Let $f(x) = xe^{3x}$

Then $f'(x) = (1)(e^{3x}) + (x)(3e^{3x})$

$= e^{3x}(1+3x)$

$= 0 \rightarrow x = -\frac{1}{3}$

A. 0

B. there are none

C. 3

D. $-1/3$

E. -3

12. Let $f(x) = 2x^3 - 3x$, $g(x) = 3x + 2\sin x$. Which one of the following statements is true?

A. Both f and g are increasing on $(-\infty, \infty)$.

B. f is increasing and g is decreasing on $(-\infty, \infty)$.

C. g is increasing on $(-\infty, \infty)$, f is not.

D. f is decreasing and g is increasing on $(-\infty, \infty)$.

E. Both f and g are decreasing on $(-\infty, \infty)$.

$f'(x) = 6x^2 - 3 = 0$

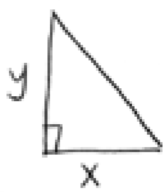
$\longrightarrow x = \pm \frac{1}{\sqrt{2}}$

	$-\infty$		$-1/\sqrt{2}$		$1/\sqrt{2}$		∞
$6x^2-3$		+	0	−	0	+	
		inc		dec		inc	

$g'(x) = 3 + 2\cos x > 0$ for all x

$\Rightarrow g$ inc for all x.

13. The lengths of the two perpendicular sides of a right triangle add up to 12 ft. What is the maximal area of the triangle?



$x + y = 12 \rightarrow y = 12 - x$

want: maximum area $A = \frac{1}{2} xy$

$A(x) = \frac{1}{2} x (12 - x) = 6x - \frac{1}{2} x^2$

$\frac{dA}{dx} = 6 - x = 0 \rightarrow x = 6$

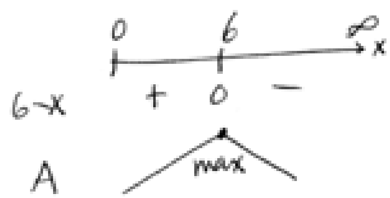
A. 12 ft^2

B. 18 ft^2

C. 72 ft^2

D. 81 ft^2

E. 144 ft^2



maximum area is $A(6)$

$A(6) = 6(6) - \frac{1}{2}(6)^2$

$= 36 - 18 = 18$

14. All antiderivatives of $1 + \cos x$ are

Let $f'(x) = 1 + \cos x$

then $f(x) = x + \sin x + C$

A. $\cos x + c$

B. $-\sin x + c$

C. $x + \sin x + c$

D. $x + x\cos x + c$

E. $x - \sin x + c$