MA161 SOLUTION KEY **EXAM 3** **November 5, 1997**

1. In a certain environment the rate of reproduction of a population of blue algae is proportional to the size of the population. It takes the population 4 hours to double in size. How long will it take for the population to triple in size?

Want to solve: $3P(0) = P(0)e^{kt}$.

Find k: $P(4) = 2P(0) = P(0)e^{4k} \rightarrow 2 = e^{4k}$

$\rightarrow \ln 2 = 4k \rightarrow k = \frac{1}{4}\ln 2$

$\therefore 3 = e^{(\frac{1}{4}\ln 2)t} \rightarrow \ln 3 = (\frac{1}{4}\ln 2)t$

$\rightarrow t = 4\frac{\ln 3}{\ln 2}$

A. 6 hrs
B. 12 hrs
C. $\frac{\ln 4}{\ln 6}$ hrs
D. $\ln 6$ hrs
[✓ E.] $\frac{4\ln 3}{\ln 2}$ hrs.

2. Find the interval or intervals where the function

$$f(x) = \frac{1}{2}x^2 + \ln(x^4)$$

is concave up.

$f'(x) = x + \frac{4x^3}{x^4} = x + \frac{4}{x}$

$f''(x) = 1 - \frac{4}{x^2} = \frac{x^2-4}{x^2} = 0 \rightarrow x = \pm 2$

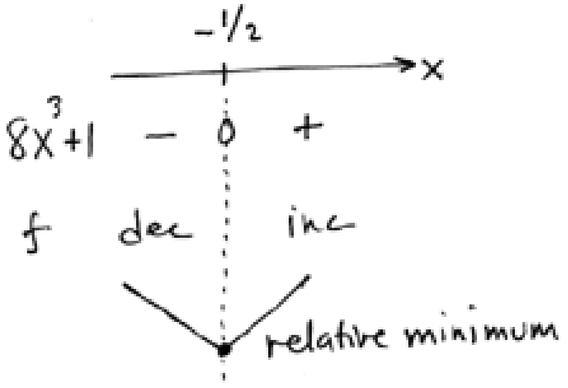
$x^2 - 4 > 0 \rightarrow x^2 > 4 \rightarrow x > 2$ or $x < -2$

A. $(2, \infty)$
B. $(-2, 2)$
[✓ C.] $(-\infty, -2)$ and $(2, \infty)$
D. $(0, 2)$
E. $(-\infty, \infty)$

3. The function $f(x) = 4x^2 - 2 - \frac{1}{x}$ has

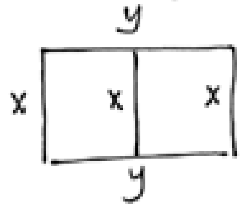
$f'(x) = 8x + \frac{1}{x^2} = \frac{8x^3+1}{x^2}$

$f'(x) = 0 \rightarrow x^3 = -\frac{1}{8} \rightarrow x = -\frac{1}{2}$



[✓ A.] a relative minimum at $x = -\frac{1}{2}$
B. a relative maximum at $x = -\frac{1}{2}$
C. a relative minimum at $x = \frac{1}{\sqrt[3]{4}}$
D. a relative maximum at $x = \frac{1}{\sqrt[3]{4}}$
E. none of the above

4. A rectangular field is to have area 2,200 square meters. Fencing is required to enclose the field and divide it in half, as shown. Fencing for the perimeter costs \$2 per meter and fencing for dividing the field in half costs \$1.50 per meter. The minimum cost is



$xy = 2200$

$C = 2(2x+2y) + 1.5(x)$ dollars

$= 5.5x + 4y$

$y = \frac{2200}{x} \rightarrow C = 5.5x + 4\left(\frac{2200}{x}\right)$

$C' = 5.5 - \frac{8800}{x^2} = \frac{5.5x^2 - 8800}{x^2}$

$C' = 0 \rightarrow x^2 = \frac{8800}{5.5} = 1600 \rightarrow x = 40.$

$C(40) = 5.5(40) + \frac{8800}{40}$

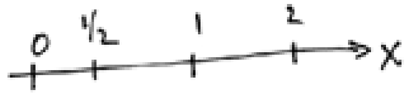
$= 220 + 220$

A. \$110
B. \$220
C. \$330
[✓ D.] \$440
E. \$550

5. If $f(x) = x^2$ and $P = \{0, \frac{1}{2}, 1, 2\}$ then the lower sum $L_f(P)$ equals

$f'(x) = 2x \rightarrow f$ increasing for $x > 0$

$\rightarrow f(x)$ minimum at left endpoint.



x^2 0 $\frac{1}{4}$ 1

$L_f(P) = 0\left(\frac{1}{2} - 0\right) + \frac{1}{4}\left(1 - \frac{1}{2}\right) + 1(2-1)$

$= 0 + \frac{1}{8} + 1 = \frac{9}{8}$

A. $\frac{5}{8}$
B. $\frac{1}{8}$
C. $\frac{1}{2}$
D. $\frac{3}{8}$
[✓ E.] none of the above

6. If $f'(x) = (x-1)(x-2)(x-3)$ then f has

[✓ A.] relative minima at 1 and 3 and a relative maximum at 2.
B. a relative minimum at 1, and relative maxima at 2 and 3.
C. relative maxima at 1 and 3 and a relative minimum at 2.
D. relative maxima at −1 and −3 and a relative minimum at −2.
E. a relative maximum at −2, and relative minima at −1 and −3.

$-\infty$ 1 2 3 $\rightarrow x$ ∞

$x-1$	−	+	+	+
$x-2$	−	−	+	+
$x-3$	−	−	−	+
$f'(x)$	−	+	−	+

relative min at $x = 1, 3$

relative max at $x = 2$

7. The maximum of x^3e^{-x} for $x > 0$ is

A. 0

B. 3

[✓ C.] $27e^{-3}$

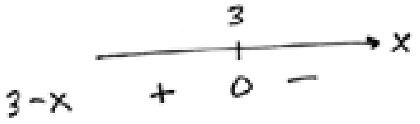
D. $\frac{1}{2}(\ln 2)^3$

E. $8e^{-2}$

$$y = x^3e^{-x}$$

$$y' = 3x^2e^{-x} - x^3e^{-x}$$

$$= x^2e^{-x}(3-x) = 0 \rightarrow x = 3$$



$$y(3) = 3^3e^{-3} = 27e^{-3}$$

8. The product of three positive numbers, two of which are known to be equal, is 64. What is the maximum and the minimum of the sum of the three numbers?

A. Min = 12, Max = 66

B. Min = 8, No Max

[✓ C.] Min = 12, No Max

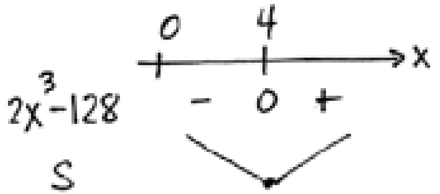
D. No Min, Max = 66

E. No Min, No Max

$$x^2y = 64 \longrightarrow y = \frac{64}{x^2}$$

$$S = 2x + y = 2x + \frac{64}{x^2}, \quad x > 0$$

$$S' = 2 - \frac{128}{x^3} = \frac{2x^3 - 128}{x^3} = 0 \rightarrow x = 4$$



$$S(4) = 2(4) + \frac{64}{4^2} = 8 + 4 = 12$$

9. What values of a and b guarantee that $\int_0^\pi f(x)dx + \int_a^b f(x)dx = \int_{-3\pi}^{\pi} f(x)dx$?

A. $a = -3\pi,\ b = \pi$

[✓ B.] $a = -3\pi,\ b = 0$

C. $a = 2\pi,\ b = -\pi$

D. $a = -2\pi,\ b = \pi$

E. $a = -4\pi,\ b = \pi$

$$\Rightarrow \int_0^\pi f(x)dx + \int_a^b f(x)dx = -\int_\pi^{-3\pi} f(x)dx$$

$$\Rightarrow \int_0^\pi f(x)dx + \int_\pi^{-3\pi} f(x)dx = -\int_a^b f(x)dx$$

$$\Rightarrow \int_0^{-3\pi} f(x)dx = \int_b^a f(x)dx$$

$$\Rightarrow a = -3\pi, \quad b = 0$$

10. If $f(x) = \begin{cases} 2x, & 0 \le x \le 2 \\ 4, & 2 \le x \le 4 \end{cases}$ then $\int_0^4 f(x)dx =$

A. 8

B. 16

[✓ C.] 12

D. 6

E. none of the above

$$\int_0^4 f(x)dx = \int_0^2 f(x)\,dx + \int_2^4 f(x)\,dx$$

$$= \int_0^2 2x\,dx + \int_2^4 4\,dx$$

$$= x^2\Big|_0^2 + 4x\Big|_2^4$$

$$= (4-0) + (16-8) = 4+8 = 12$$

11. The inflection points of $f(x) = \frac{5}{x} - \frac{5}{x^3}$ are

A. $-1, 1$

B. $-1,\ \sqrt{6}$

[✓ C.] $-\sqrt{6},\ \sqrt{6}$

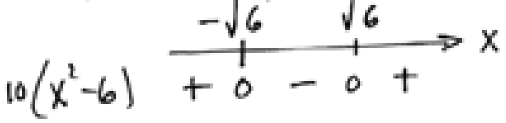
D. $-\sqrt{6},\ \sqrt{3}$

E. $-\sqrt{3},\ \sqrt{3}$

$$f'(x) = -\frac{5}{x^2} + \frac{15}{x^4}$$

$$f''(x) = \frac{10}{x^3} - \frac{60}{x^5} = \frac{10x^2-60}{x^5}$$

$$f''(x) = 0 \rightarrow x = \pm\sqrt{6}$$



$\Rightarrow$ inflection pts at $-\sqrt{6}, \sqrt{6}$

12. $\lim\limits_{x\to\infty} \frac{3x^3+17x^2}{4x^3-18x^2} =$

A. ∞

B. $-\infty$

C. $-\frac{17}{18}$

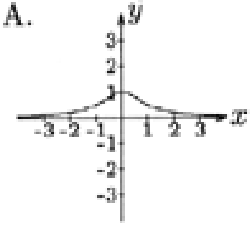
[✓ D.] $\frac{3}{4}$

E. none of the above

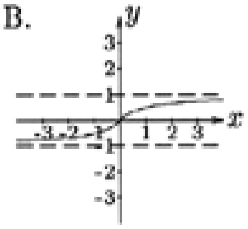
$$= \lim_{x\to\infty} \frac{3+\frac{17}{x}}{4-\frac{18}{x}} = \frac{3+0}{4-0} = \frac{3}{4}$$

13. Which of the sketches could be the graph of $f(x) = \frac{x}{x^2+1}$.

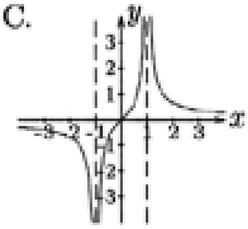
A.



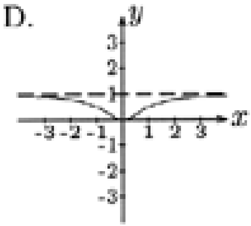
B.



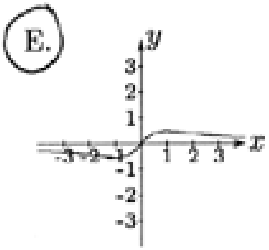
C.



D.



[✓ E.]



f(x) has same sign as x.

$$\lim_{\substack{x\to\infty \\ x\to-\infty}} \frac{x}{x^2+1} = 0$$

→ horizontal asymptote is $y=0$.

14. $\lim_{x\to\infty} \frac{\sin(x^2)}{x} =$

$$\lim_{x\to\infty} \left(\frac{1}{x}\right)\left(\sin x^2\right) = 0$$

A. 2

[✓ B.] 0

C. ∞

D. 1

E. $-\infty$