MA161 **EXAM 1** **September 16, 1998**

Name: Solution Key ID #: ______

Section # ______ TA's Name: ______

Instructions:

1. Fill in your name, student ID number and division and section number on the mark–sense sheet. Also fill out the information requested above.
2. This booklet consists of 8 pages. There are 14 questions, each worth 7 points.
3. Mark your answers on the mark–sense sheet with a #2 pencil. Please show your work in this booklet.
4. No books, notes or calculator may be used.
5. When you are finished with the exam, please hand this booklet and the mark–sense sheet, in person, to your instructor.

1. If $|2x-1| \leq 2$ then

   A. $-\frac{1}{2} \leq x \leq \frac{1}{2}$
   B. $-\frac{1}{2} \leq x \leq \frac{3}{2}$
   C. $-1 \leq x \leq 3$
   D. $-2 \leq x \leq 6$
   E. $-\frac{3}{2} \leq x \leq \frac{1}{2}$

$\rightarrow -2 \leq 2x-1 \leq 2$

$\rightarrow -1 \leq 2x \leq 3$

$\rightarrow -\frac{1}{2} \leq x \leq \frac{3}{2}$

2. Find an equation for the line through $(4,2)$ and perpendicular to the line with equation $y=4$.

   A. $x=4$
   B. $y=2$
   C. $y-4x+14=0$
   D. $4y-x-4=0$
   E. None of the above

$y=4$ is horizontal line.

Perpendicular line is vertical.

$\Rightarrow x=4$

3. A ball is thrown straight up from the top of a 120 foot tall building. It hits the ground 3 seconds later. What was its initial velocity? ($h(t) = -16t^2 + v_0 t + h_0$).

A. 40

B. -40

C. 10

D. -8

E. 8

$$h(t) = -16t^2 + v_0 t + 120$$

$$h(3) = 0 = -16(3)^2 + v_0(3) + 120$$

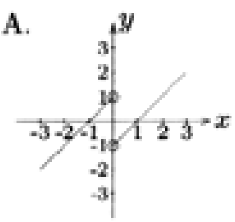
$$\longrightarrow 0 = -144 + 3v_0 + 120$$

$$\longrightarrow 24 = 3v_0$$

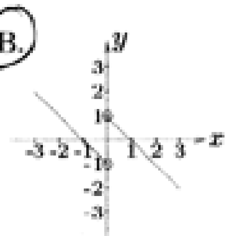
$$\longrightarrow v_0 = 8$$

4. The graph of the function $f(x) = \dfrac{|x|}{x} - x$ looks most like

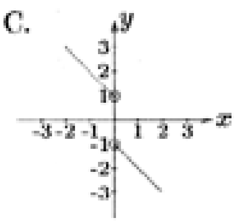
A.



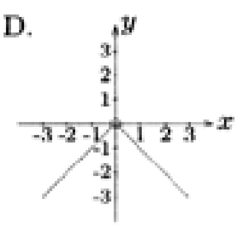
B.



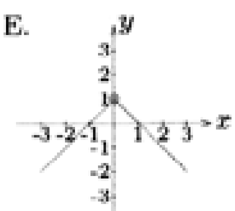
C.



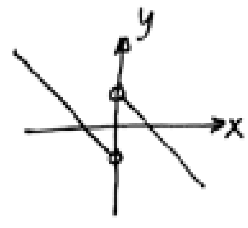
D.



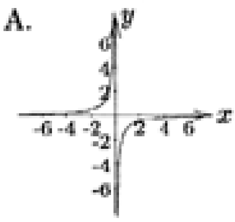
E.

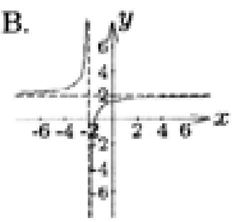


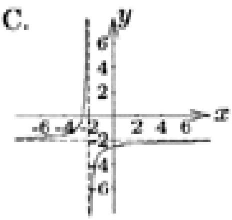
$$\frac{|x|}{x} - x = \begin{cases} \frac{x}{x} - x = 1 - x & \text{if } x > 0 \\ \frac{-x}{x} - x = -1 - x & \text{if } x < 0 \end{cases} \Longrightarrow$$

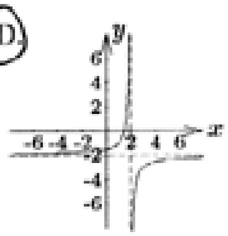


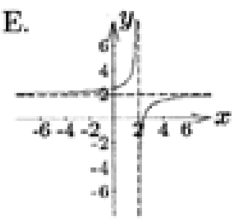
5. The graph of $y + 2 = \dfrac{1}{2 - x}$ looks most like

A.


B.


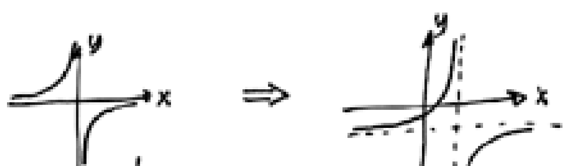
C.


D.


E.


$\Rightarrow y - (-2) = \dfrac{-1}{x - (2)}$

translate graph of $y = \dfrac{-1}{x}$ 2 units to right and 2 units down.



$y = \dfrac{-1}{x}$

6. Let $f(x) = \dfrac{1}{x-1}$ and $g(x) = \dfrac{1}{1-x}$. Find the rule for and the domain of $f \circ g$.

A. $f \circ g(x) = \dfrac{x-1}{x}$, $x \neq 0, 1$

B. $f \circ g(x) = \dfrac{1-x}{x}$, $x \neq 0, 1$

C. $f \circ g(x) = \dfrac{x-1}{x}$, $x = 0$

D. $f \circ g(x) = \dfrac{x}{x+1}$, $x \neq -1$

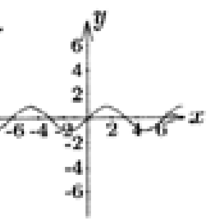
E. $f \circ g(x) = -1$, $x \neq 1$

$$(f \circ g)(x) = f(g(x))$$
$$= f\left(\frac{1}{1-x}\right)$$
$$= \frac{1}{\frac{1}{1-x} - 1}$$
$$= \frac{1}{\frac{1-(1-x)}{1-x}}$$
$$= \frac{1-x}{x} \quad x \neq 0, \ x \neq 1$$

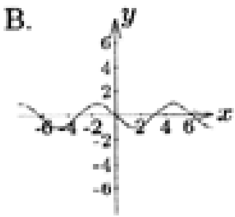
domain of g is $x \neq 1$

7. A sketch of the graph of $y = \sin(\pi/2 - x)$ looks like

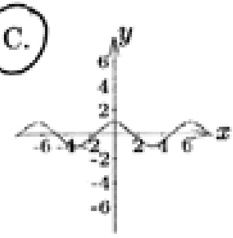
A.



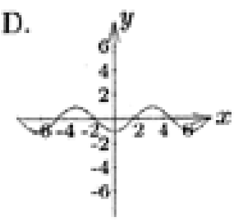
B.



C.



D.



E. None of A, B, C or D.

$$y = \sin\left(\frac{\pi}{2} - x\right)$$
$$= \sin\left(-\left(x - \frac{\pi}{2}\right)\right)$$
$$= -\sin\left(x - \frac{\pi}{2}\right) \quad \text{since sine is odd fcn.}$$

translate graph of $y = -\sin x$, $\frac{\pi}{2}$ units to right

8. Solve: $\ln(x) + \ln(x+5) - \ln(x^2) = \ln 13$.

A. $x = 0, 5/12$

B. $x = \sqrt{13}, 1$

C. $x = 0$

D. $x = 0, -5/12$

E. $x = 5/12$

$$\Rightarrow \ln \frac{x(x+5)}{x^2} = \ln 13$$
$$\Rightarrow \frac{x(x+5)}{x^2} = 13$$
$$\Rightarrow x^2 + 5x = 13x^2$$
$$\Rightarrow -12x^2 + 5x = 0$$
$$\Rightarrow x(-12x + 5) = 0$$
$$\Rightarrow x = 5/12$$

Note: x cannot equal zero since $\ln(0)$ not defined. (1st term on left of original eqn.)

9. Simplify $\frac{4^{\sqrt{2}}8^{(\sqrt{2}-1)}}{2^{5\sqrt{2}}}$.

$$= \frac{(2^2)^{\sqrt{2}}(2^3)^{\sqrt{2}-1}}{2^{5\sqrt{2}}}$$

A. 1

B. 1/2

C. $\frac{1}{\sqrt{2}}$

D. $\frac{1}{4}$

E. $\frac{1}{8}$

$$= 2^{2\sqrt{2}+3\sqrt{2}-3-5\sqrt{2}}$$

$$= 2^{-3}$$

$$= \frac{1}{8}$$

10. Solve the inequality $\tan x \leq -1$ for x in $[0, \pi]$.

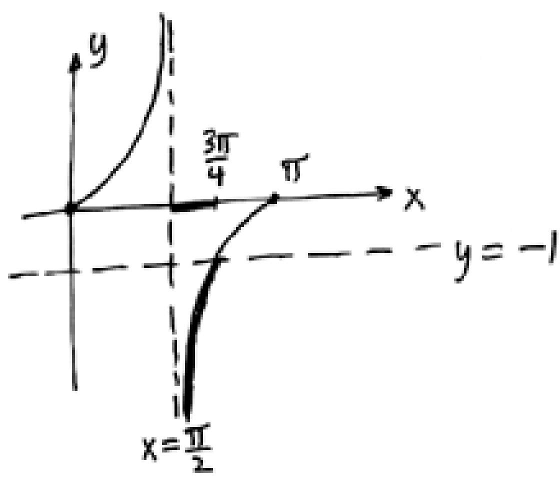
A. $\frac{\pi}{4} \leq x < \frac{\pi}{2}$

B. $\frac{\pi}{2} < x \leq \pi$

C. $\frac{\pi}{2} < x \leq \frac{3\pi}{4}$

D. $\frac{3\pi}{4} \leq x \leq \pi$

E. there are no solutions



11. Evaluate $\lim_{x \to -1} \frac{x^2 + 3x + 2}{x^2 - 1}$. $= \frac{1-3+2}{1-1} = \frac{0}{0}$

A. 2

B. $\frac{3}{2}$

C. $\frac{1}{2}$

D. $-\frac{1}{2}$

E. the limit does not exist

$$= \lim_{x \to -1} \frac{\cancel{(x+1)}(x+2)}{\cancel{(x+1)}(x-1)} = \frac{-1+2}{-1-1} = \frac{1}{-2}$$

12. What value of b makes the following function continuous at $x = 0$?

$$f(x) = \begin{cases} \dfrac{\sin^2 x - x}{x \cos^2 x} & \text{for } x \neq 0 \\ b & \text{for } x = 0 \end{cases}$$

A. 1

B. -1

C. 0

D. 2

E. No value of b makes f continuous at $x = 0$.

f continuous at $x=0$ if $\lim_{x \to 0} f(x) = f(0)$

Clearly, $f(0) = b$. Hence we want $b = \lim_{x \to 0} f(x)$.

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{\sin^2 x - x}{x \cos^2 x} = \lim_{x \to 0} \left(\frac{\sin^2 x}{x \cos^2 x} - \frac{x}{x \cos^2 x} \right)$$

$$= \lim_{x \to 0} \left(\left(\frac{\sin x}{x}\right)\left(\frac{\sin x}{\cos^2 x}\right) - \left(\frac{x}{x}\right)\left(\frac{1}{\cos^2 x}\right) \right) = (1)(0) - (1)(1) = -1$$

13. Let $f(x) = \dfrac{x-1}{x+1}$ and $g(x) = \dfrac{x^3-1}{x^2-2x+1}$. Which one of the following statements is true?

A. Neither f nor g is continuous at $x = -1$

B. Both f and g are continuous at $x = -1$

C. Only f is continuous at $x = -1$

D. Only g is continuous at $x = -1$

E. The above statements are all false.

$f(-1) = \frac{-2}{0}$, undefined. Therefore f not continuous at $x=-1$

$$g(-1) = \frac{-1-1}{1+2+1} = \frac{-2}{4} = -\frac{1}{2}$$

$$\lim_{x\to -1} g(x) = \frac{(-1)^3-1}{(-1)^2-2(-1)+1} = \frac{-2}{4} = -\frac{1}{2}$$

$\rightarrow$ g is cont. at $x=-1$

14. Evaluate $\lim_{x\to 0} x\left[1+\sin\frac{1}{x}\right]$.

A. -1

B. 0

C. 1

D. 2

E. the limit does not exist.

Note: $\lim_{x\to 0}\left[1+\sin\frac{1}{x}\right]$ does not exist.

However,

$$-1 \le \sin\frac{1}{x} \le 1$$

$$\longrightarrow \quad 0 \le 1+\sin\frac{1}{x} \le 2$$

$$\xrightarrow{\text{(assume } x>0\text{)}} \quad 0 \le x\left(1+\sin\frac{1}{x}\right) \le 2x$$

and $\lim_{x\to 0^+} 0 = \lim_{x\to 0^+} 2x = 0$. Therefore by squeeze theorem, $\lim_{x\to 0^+} x\left(1+\sin\frac{1}{x}\right) = 0$. A similar argument shows that $\lim_{x\to 0^-} x\left(1+\sin\frac{1}{x}\right) = 0$.