MA161 **EXAM 2** **October 15, 1998**

Name: SOLUTION KEY

I.D.#: ______________________

Recitation Instructor: ______________________ **Time of Recitation** ________

Lecturer: ______________________ **Section#:** ______________________

Instructions:

(1) Fill in your name, student ID number and division and section number on the mark-sense sheet. Also fill out the information requested above.
(2) This booklet consists of 6 pages. There are 14 questions, each worth 7 points.
(3) Mark your answers on the mark-sense sheet. Please show your working in this booklet.
(4) No books, notes or calculators may be used.
(5) When you are finished with the exam hand this booklet and the mark-sense sheet, in person, to your instructor.

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1. If $f(t) = \dfrac{t^2}{1+t^3}$, $f'(t) =$

$$f'(t) = \frac{(2t)(1+t^3) - (t^2)(3t^2)}{(1+t^3)^2}$$

$$= \frac{2t + 2t^4 - 3t^4}{(1+t^3)^2}$$

$$= \frac{2t - t^4}{(1+t^3)^2}$$

A. $\dfrac{2t - 3t^2}{(1+t^3)^2}$

B. $\dfrac{1 + t^2 + t^3}{(1+t^3)^2}$

C. $\dfrac{2t - t^4}{(1+t^3)^2}$

D. $\dfrac{2t - 5t^4}{(1+t^3)^2}$

E. $\dfrac{2t}{(1+t^3)^2}$

2. If $f(t) = \cos(\ln(3t^2))$, $f'(t) =$

$$f'(t) = \left(-\sin(\ln(3t^2))\right)\left(\frac{d}{dt}\ln(3t^2)\right)$$

$$= -\sin(\ln(3t^2))\left(\frac{1}{3t^2}\cdot 6t\right)$$

$$= -\frac{2\sin(\ln(3t^2))}{t}$$

A. $\dfrac{-2\sin(\ln(3t^2))}{t}$

B. $-\sin\left(\dfrac{1}{3t^2}\right)$

C. $\dfrac{-\sin(\ln(3t^2))}{3t^2}$

D. $-\dfrac{1}{\sin(3t^2)}$

E. $\tan(3t^2)$

3. Given that $f(2) = 3$, $f(8) = 4$, $f'(2) = 5$, $f'(8) = -1$ and $f''(2) = 6$, evaluate

$$\frac{d}{dx}[f(x^3) \cdot f(x)]$$

at $x = 2$.

$$\frac{d}{dt}\left[f(x^3)\cdot f(x)\right]$$

$$= f'(x^3)\cdot 3x^2 \cdot f(x) + f(x^3)\cdot f'(x)$$

$$x=2 \rightarrow f'(8)\cdot 12 \cdot f(2) + f(8)\cdot f'(2)$$

$$= (-1)(12)(3) + (4)(5)^2 = -36 + 20 = -16$$

A. 17

B. 8

C. 0

D. -5

E. -16

4. If $g(x) = -e^{-3x} + x^{21} - x^2$ then the twenty-third derivative of g, $g^{(23)}(x) =$

try to find a pattern: consider first few derivatives:

$$\left.\begin{aligned} g'(x) &= 3e^{-3x} + 21x^{20} - 2x \\ g''(x) &= -3^2 e^{-3x} + 21\cdot 20 x^{19} - 2 \\ g'''(x) &= 3^3 e^{-3x} + 21\cdot 20\cdot 19 x^{18} \\ g^{(4)}(x) &= -3^4 e^{-3x} + 21\cdot 20\cdot 19\cdot 18 x^{17} \\ &\vdots \end{aligned}\right] \Rightarrow g^{(23)}(x) = 3^{23} e^{-3x}$$

A. $3^{23}e^{-3x}$

B. $-e^{-3x}$

C. $-3^{23}e^{-3x} + 21$

D. 0

E. $-3^{23}e^{-3x}$

5. If $x^3 + xy^2 + 3y^3 = \pi^{\frac{1}{2}}$ then $\dfrac{dy}{dx} =$

Differentiate w.r.t. $x \Rightarrow$

$$3x^2 + y^2 + 2xy\frac{dy}{dx} + 9y^2\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx}(2xy + 9y^2) = -3x^2 - y^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{-3x^2 - y^2}{2xy + 9y^2}$$

A. $\dfrac{-x^2}{2xy + 9y^2}$

B. $\dfrac{\pi^{\frac{1}{2}} - x^3}{xy + 3y^2}$

C. $-(3x + y^2)$

D. $\dfrac{-3x^2 - y^2}{2xy + 9y^2}$

E. $\dfrac{\pi^{\frac{1}{2}}}{x^3 + x^2y + 3y^2}$

6. A spherical balloon is inflated in such a way that after t seconds $V = 36\pi\sqrt{t}$ cubic centimeters. How fast is the radius of the balloon changing when $t = 64$?

want: $\dfrac{dr}{dt}$ when $t = 64$.

$$V = \frac{4}{3}\pi r^3 \longrightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\longrightarrow \frac{dr}{dt} = \frac{dV}{dt}\left(\frac{1}{4\pi r^2}\right)$$

Note: $t = 64 \rightarrow V = 36\pi\sqrt{64} = 36\pi 8 = \frac{4}{3}\pi r^3$

$\longrightarrow r^3 = 36\pi 8\left(\frac{3}{4\pi}\right) = 36\cdot 2\cdot 3 = 6^3 \rightarrow r = 6$

also $\dfrac{dV}{dt} = 36\pi\dfrac{1}{2\sqrt{t}} = 36\pi\dfrac{1}{2\sqrt{64}} = \dfrac{36\pi}{2\cdot 8} = \dfrac{18\pi}{8} = \dfrac{9\pi}{4} \Rightarrow \dfrac{dr}{dt} = \dfrac{9\pi}{4}\left(\dfrac{1}{4\pi 6^2}\right) = 1/64.$

A. 1

B. $\dfrac{1}{16}$

C. $\dfrac{1}{32}$

D. $\dfrac{1}{64}$

E. $\dfrac{1}{128}$

7. The edges of a cube are increasing at the rate of 4 inches/min. At what rate is the volume of the cube increasing when the volume is 8 cubic inches?

Know: $\frac{de}{dt} = 4$ want: $\frac{dV}{dt}$ when $V=8$.

$V = e^3 \longrightarrow \frac{dV}{dt} = 3e^2 \frac{de}{dt}$

$\longrightarrow \frac{dV}{dt} = 3 \cdot 2^2 \cdot 4$ $(V=8 \to e=2)$

$= 48$

A. 12 in.^{3}/min.
B. 16 in.^{3}/min.
C. 8π in.^{3}/min.
D. 32 in.^{3}/min.
E. 48 in.^{3}/min.

8. Use the fact that $(16)^{\frac{1}{4}} = 2$ and use linear approximation to approximate $(14)^{\frac{1}{4}}$.

Let $f(x) = x^{1/4}$ $\left(\to f'(x) = \frac{1}{4} x^{-3/4}\right)$

Note: $(14)^{1/4} = (16-2)^{1/4}$

$\approx f(16) + f'(16)\,dx$

$= 16^{1/4} + \frac{1}{4}(16)^{-3/4}(-2)$

$= 2 + \frac{1}{4}\left(\frac{1}{8}\right)(-2)$

$= 2 - \frac{1}{16}$

A. $2 - \frac{1}{8}$
B. $2 - \frac{1}{16}$
C. $2 - \frac{1}{32}$
D. 2
E. $2 + \frac{1}{32}$

9. The critical numbers of $f(x) = \frac{200}{x} + 2x - 50$ are

$f'(x) = \frac{-200}{x^2} + 2 = \frac{-200 + 2x^2}{x^2}$

$f'(x)$ does not exist if $x=0$ (but $f(0)$ does not exist)

$\Longrightarrow$ $x=0$ is not a critical number.

$f'(x) = 0 \to -200 + 2x^2 = 0 \to x^2 = 100 \to x = \pm 10$

A. 5, 0, 20
B. 5, 20
C. -10, 10
D. -10, 0, 10
E. There are none

10. Find all extreme values (if any) of $f(x) = x^2 + \dfrac{16}{x}$ on the interval $[1, 4]$.

A. max. value = 20; min. value = 17

B. max. value = 20; min. value = 12

C. max, value = 18; min. value = 8

D. no max. value; min. value = 17

E. no max. value; no min. value

$f'(x) = 2x - \frac{16}{x^2} = \frac{2x^3 - 16}{x^2}$

$f'(x) = 0 \rightarrow x = \pm 2$

$f'(x)$ does not exist $\rightarrow x = 0$

only critical pt. in $[1,4]$ is $x = 2$.

x	$x^2 + \frac{16}{x}$
1	$1 + 16 = 17$
2	$4 + \frac{16}{2} = 12 \longleftarrow$ MIN
4	$16 + \frac{16}{4} = 20 \longleftarrow$ MAX

11. A number c in the interval $(0, 2)$ for which the line tangent to the graph of $y = x^3 - x^2$ at $x = c$ is parallel to the line through $(0, 0)$ and $(2, 4)$ is

A. 1

B. $\frac{4}{3}$

C. $\frac{2 + \sqrt{10}}{6}$

D. $\frac{1 + \sqrt{7}}{3}$

E. $\frac{2 + \sqrt{40}}{6}$

This is a question about the Mean Value Theorem.

$y'(c) = \frac{y(2) - y(0)}{2 - 0} = \frac{4 - 0}{2 - 0} = 2$

$y'(x) = 3x^2 - 2x \rightarrow y'(c) = 3c^2 - 2c$

Solve for c: $3c^2 - 2c = 2$

$\rightarrow 3c^2 - 2c - 2 = 0$

$\rightarrow c = \frac{2 \pm \sqrt{4 - 4(3)(-2)}}{2(3)}$

$= \frac{2 \pm \sqrt{4 + 24}}{2(3)} = \frac{2 \pm \sqrt{28}}{2 \cdot 3}$

$= \frac{2 \pm 2\sqrt{7}}{2 \cdot 3} = \frac{1 \pm \sqrt{7}}{3} = \frac{1 + \sqrt{7}}{3}$ on $(0,2)$

12. Suppose you have a cache of a radioactive substance whose half-life is 250 years. How long will you have to wait for $\frac{4}{5}$ of it to decay (i.e., $\frac{1}{5}$ to remain)?

half-life $= 250 \rightarrow 250k = \ln\frac{1}{2} \rightarrow k = \frac{1}{250}\ln\frac{1}{2}$

$\Rightarrow A(t) = A(0)\, e^{\left(\frac{1}{250}\ln\frac{1}{2}\right)t}$

Solve for t: $A(t) = \frac{1}{5}A(0) = A(0)\, e^{\left(\frac{1}{250}\ln\frac{1}{2}\right)t}$

$\rightarrow \frac{1}{5} = e^{\left(\frac{1}{250}\ln\frac{1}{2}\right)t}$

$\rightarrow \ln\frac{1}{5} = \left(\frac{1}{250}\ln\frac{1}{2}\right)t$

$\rightarrow t = 250\,\frac{\ln\frac{1}{5}}{\ln\frac{1}{2}} = 250\,\frac{\ln 5^{-1}}{\ln 2^{-1}}$

$= 250\,\frac{-\ln 5}{-\ln 2} = 250\,\frac{\ln 5}{\ln 2}$

A. $250\frac{\ln 5}{\ln 2}$ years

B. $250\frac{\ln 2}{\ln 5}$ years

C. $250\ln\left(\frac{2}{5}\right)$ years

D. $250\ln\left(\frac{5}{2}\right)$ years

E. 50 years

13. Let $f(x) = \frac{5}{x}$ and $g(x) = x^3$. Then

$f'(x) = \frac{-5}{x^2} < 0$ if $x \neq 0$

$g'(x) = 3x^2 > 0$ for $x \neq 0$

A. both f and g are increasing on $(0, \infty)$

B. both f and g are decreasing on $(0, \infty)$

C. f is increasing and g is decreasing on $(0, \infty)$

D. f is decreasing and g is increasing on $(0, \infty)$

E. none of the above is true.

14. The function $h(x) = 4x^3 - 3x^4$ has

$h'(x) = 12x^2 - 12x^3$

$= 12x^2(1 - x)$

A. no relative extrema

B. one relative extremum

C. two relative extrema

D. three relative extrema

E. four relative extrema.

x	$-\infty$ to 0	0 to 1	1 to ∞
$12x^2$	+	+	+
$1-x$	+	−	−
$h'(x)$	+	−	−

h 

h has one relative max and no relative min.