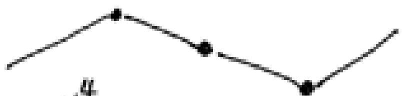
MA 161 SOLUTION KEY **EXAM 3** **November 10, 1998**

1. Let $f'(x) = (x-1)(x-2)^2(x-3)e^{x^4 \cos x}$. Consider the following statements.
   I. f has a relative maximum at $x = 1$.
   II. f has a relative minimum at $x = 2$. Then

x: $-\infty$ … ∞	$(-\infty, 1)$	$(1, 2)$	$(2, 3)$	$(3, \infty)$
$x-1$	−	+	+	+
$(x-2)^2$	+	+	+	+
$x-3$	−	−	−	+
$f'(x)$	+	−	−	+

f 

note: $e^{x^4 \cos x} > 0$.

A. I and II are both true.

B. I is true, II is false.

C. I is false, II is true.

D. I and II are both false.

E. There is insufficient information to decide whether these statements are true or false.

2. Let $f'(x) = (x-3)x^2$. Then the graph of $y = f(x)$ is concave up when

$$f''(x) = (1)(x^2) + (x-3)(2x)$$
$$= 3x^2 - 6x = 3x(x-2)$$

x: $-\infty$ … ∞	$(-\infty, 0)$	$(0, 2)$	$(2, \infty)$
$3x$	−	+	+
$x-2$	−	−	+
$f''(x)$	+	−	+
concave	up	down	up

A. $x > \sqrt{2}$ and when $x < -\sqrt{2}$

B. $x > 2$ and when $x < 0$

C. $-\sqrt{2} < x < \sqrt{2}$

D. $0 < x < 2$

E. The graph is never concave up.

3. Find the point on the graph of $y = \sqrt{x}$ that is closest to the point $(4, 0)$. The distance from this point to $(4, 0)$ is

$$D = \sqrt{(x-4)^2 + (\sqrt{x} - 0)^2} = \sqrt{x^2 - 7x + 16}$$

$$\frac{dD}{dx} = \frac{2x-7}{2\sqrt{x^2-7x+16}} = 0 \longrightarrow x = \frac{7}{2}$$

x		$7/2$	
$2x-7$	−	0	+
D	dec		inc

min

$$D\left(\tfrac{7}{2}\right) = \sqrt{\left(\tfrac{7}{2}-4\right)^2 + \left(\sqrt{\tfrac{7}{2}}\right)^2}$$
$$= \sqrt{\tfrac{1}{4} + \tfrac{7}{2}} = \sqrt{\tfrac{15}{4}}$$
$$= \frac{\sqrt{15}}{2}$$

A. $\frac{\sqrt{18}}{2}$

B. $\frac{\sqrt{17}}{2}$

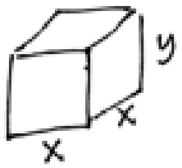
C. 2

D. $\frac{\sqrt{15}}{2}$

E. $\frac{\sqrt{14}}{2}$

4. A shed is to have a square base, a flat, horizontal roof, and a volume of 800 cubic feet. The floor costs \$6 per square foot, the roof \$2 per square foot and the walls \$5 per square foot. The cost of the cheapest such shed will be

A. \$2400
B. \$2000
C. \$1600
D. \$1200
E. \$800



$800 = x^2 y \longrightarrow y = \frac{800}{x^2}$

$C = 6x^2 + 2x^2 + 5(4xy)$

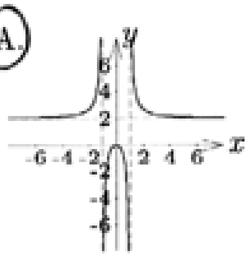
$= 8x^2 + 20xy$

substituting $\frac{800}{x^2}$ for $y \Rightarrow C = 8x^2 + 20x\left(\frac{800}{x^2}\right)$

$C = 8x^2 + \frac{16,000}{x} \Rightarrow C' = 16x - \frac{16,000}{x^2} = \frac{16x^3 - 16,000}{x^2}$

$C' = 0 \rightarrow x = 10$. $C(10) = 8(10)^2 + \frac{16,000}{10} = 800 + 1,600 = 2400$.

5. The graph of the function $\dfrac{2x^2}{x^2-1}$ looks most like

A.


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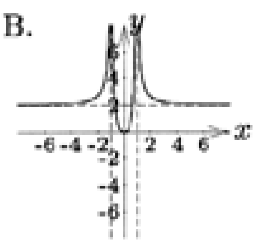

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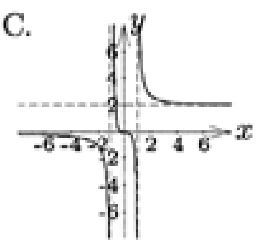

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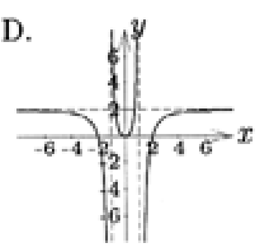

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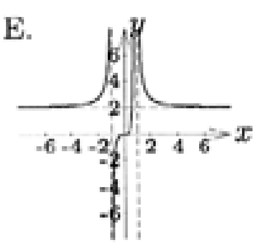


horizontal asymptote: $y = 2$

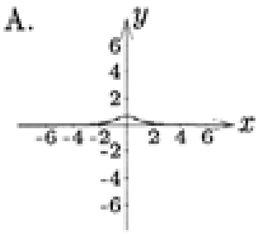
vertical asymptotes: $x = \pm 1$

sign of $f(x)$

	$-\infty$ to -1	-1 to 0	0 to 1	1 to ∞
$2x^2$	+	+	+	+
x^2-1	+	−	−	+
$f(x)$	+	−	−	+

$\Rightarrow$ answer must be A.

6. The graph of $\ln(e^{x^3}+1)$ looks most like

A.


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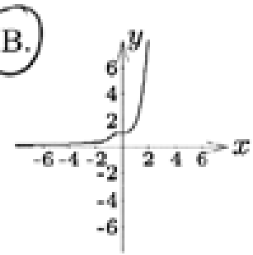

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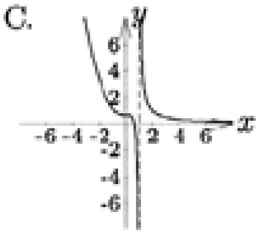

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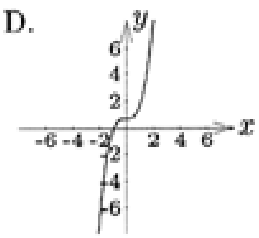

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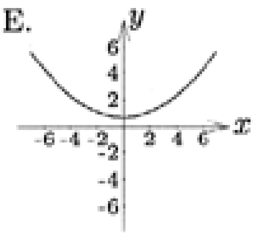


$\lim_{x\to\infty} e^{x^3}+1 = \infty$; $\ln \to \infty$

$\lim_{x\to-\infty} e^{x^3}+1 = 1$; $\ln \to 0$

7. $\lim_{x\to\infty} \sqrt{4x^2-x} - 2x =$

$\left(\sqrt{4x^2-x} - 2x\right)\dfrac{\sqrt{4x^2-x}+2x}{\sqrt{4x^2-x}+2x}$

$= \dfrac{4x^2-x-4x^2}{\sqrt{4x^2-x}+2x} = \dfrac{-x}{\sqrt{4x^2-x}+2x}$

$\lim_{x\to\infty} \dfrac{-1}{\sqrt{4-\frac{1}{x}}+2} = \dfrac{-1}{\sqrt{4}+2} = -\dfrac{1}{4}$

A. -1

B. $\dfrac{-1}{4}$

C. 0

D. 2

E. The limit does not exist

8. Let $F(x) = \int_0^x (t^2+\sin^2 t)dt$. Consider the following statements:

I. $F(x)$ is everywhere increasing

II. $F(0)=0$

III. $F(-1)=F(1)$

Then

I. $F'(x) = x^2+\sin^2 x \geq 0 \Rightarrow T$

II $F(0) = \int_0^0 \cdots = 0 \Rightarrow T$

III $t^2+\sin^2 t$ is an even fcn.

$\Rightarrow \int_{-1}^0 \cdots = \int_0^1 \cdots$

$\Rightarrow -F(-1) = F(1) \Rightarrow F$

A. I, II, III are true

B. I and II are true; III is false

C. I is true; II and III are false

D. II is true; I and III are false

E. III is true, I and II are false

9. What value of a makes the following equation true for every continuous function f?

$$\int_1^4 f(t)dt - \int_a^4 f(t)dt = \int_1^{4-a} f(t)dt.$$

$\Rightarrow \int_1^4 f(t)dt + \int_4^a f(t)\,dt = \int_1^{4-a} f(t)\,dt$

$\Rightarrow \int_1^a f(t)\,dt = \int_1^{4-a} f(t)\,dt$

$\Rightarrow a = 4-a$

$\Rightarrow 2a = 4 \Rightarrow a = 2$

A. 1
B. 2
C. 3
D. 4
E. No value of a

10. $\dfrac{d}{dx}\displaystyle\int_{x^2}^{x^3} \sin(e^t)dt =$

$\dfrac{d}{dx}\left(-\int_a^{x^2} \sin e^t\,dt + \int_a^{x^3} \sin e^t\,dt\right)$

$= \left(-\sin e^{x^2}\right)(2x) + \left(\sin e^{x^3}\right)(3x^2)$

A. $\sin(e^{x^3}) - \sin(e^{x^2})$
B. $\cos(e^{x^3}) - \cos(e^{x^2})$
C. $(3x^2 - 2x)\sin(e^x)$
D. $3x^2\sin(e^{x^3}) - 2x\sin(e^{x^2})$
E. $3x^2e^{x^3}\cos(e^{x^3}) - 2xe^{x^2}\cos(e^{x^2})$

11. $\displaystyle\int_3^8 \frac{|x-5|}{3}dx = \int_3^5 \frac{|x-5|}{3}dx + \int_5^8 \frac{|x-5|}{3}dx$

$= \displaystyle\int_3^5 \frac{-x+5}{3}dx + \int_5^8 \frac{x-5}{3}dx$

$= \frac{1}{3}\left(\left(-\frac{1}{2}x^2+5x\right)\Big|_3^5 + \left(\frac{1}{2}x^2-5x\right)\Big|_5^8\right)$

$= \frac{1}{3}\left(\left(\frac{25}{2}\right)-\left(-\frac{9}{2}+15\right) + (32-40)-\left(-\frac{25}{2}\right)\right)$

$= \frac{1}{3}\left(\frac{25}{2} - \frac{21}{2} - 8 + \frac{25}{2}\right) = \frac{1}{3}\left(\frac{25-21-16+25}{2}\right) = \frac{13}{6}$

A. $\frac{5}{6}$
B. 2
C. $\frac{13}{6}$
D. 4
E. 6

12. $\displaystyle\int_1^{e^2} \frac{\ln\sqrt{x}}{x}\,dx =$

A. -2
B. -1
C. 0
D. 1
E. 2

Let $u = \ln\sqrt{x} \rightarrow du = \frac{1}{\sqrt{x}}\cdot\frac{1}{2\sqrt{x}}\,dx = \frac{1}{2}\cdot\frac{1}{x}\cdot dx$

$\rightarrow \frac{1}{x}dx = 2\,du$

$\Rightarrow \displaystyle\int_0^1 u\,2\,du = u^2\Big|_0^1 = 1$

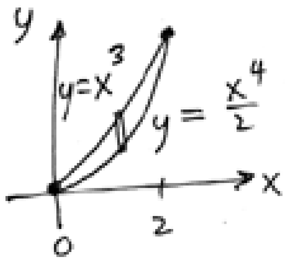
13. The area between the curves $y = x^3$ and $y = \dfrac{x^4}{2}$ is

A. $\displaystyle\int_0^1 \left(x^3 - \frac{x^4}{2}\right) dx$
B. $\displaystyle\int_0^1 \left(\frac{x^4}{2} - x^3\right) dx$
C. $\displaystyle\int_0^2 \left(x^3 - \frac{x^4}{2}\right) dx$
D. $\displaystyle\int_0^2 \left(\frac{x^4}{2} - x^3\right) dx$
E. None of the above.

intersection: $x^3 = \frac{x^4}{2} \rightarrow 2x^3 = x^4$

$\rightarrow x^4 - 2x^3 = 0 \rightarrow x^3(x-2) = 0$

$\rightarrow x = 0, 2$



area $= \displaystyle\int_0^2 \left(x^3 - \frac{x^4}{2}\right)dx$

14. $\displaystyle\int_{\pi/2}^{\pi/6} \frac{\cos t}{\sin^3 t}\,dt =$

A. $\frac{15}{4}$
B. $\frac{3}{2}$
C. 1
D. -1
E. $\frac{-3}{2}$

$u = \sin t \rightarrow du = \cos t\,dt$

$\displaystyle\int_{\sin\frac{\pi}{2}}^{\sin\frac{\pi}{6}} u^{-3}\,du = \frac{u^{-2}}{-2}\Big|_1^{\frac{1}{2}}$

$= -\frac{1}{2}\left(\left(\frac{1}{2}\right)^{-2} - (1)^{-2}\right) = -\frac{1}{2}(4-1) = -\frac{3}{2}$