MATH 161 – FALL 1999 – FIRST EXAM

September 16, 1999

STUDENT NAME: Solution Key

STUDENT ID NUMBER: ____________________

RECITATION INSTRUCTOR: ____________________

INSTRUCTIONS:

1. This test booklet has 5 pages including this one.
2. Fill in your name, your student ID number, and your recitation instructor's name above.
3. Use a number 2 pencil on the mark-sense sheet (answer sheet).
4. On the mark-sense sheet, fill in the recitation instructor's name and the course number.
5. Fill in your name and student ID number, blacken the appropriate spaces, and sign the mark-sense sheet.
6. Mark the the division and section number of your class and blacken the corresponding circles, including the circles for the zeros. If you do not know your division and section number ask your instructor.
7. There are 12 questions, each worth 8 points. Blacken your choice of the correct answer in the spaces provided for questions 1-12. Turn in BOTH the answer sheet and the question sheets to your instructor when you are finished.
8. No books, notes or calculators may be used.

Useful formulas:

$\sin(x+y) = \sin x \cos y + \sin y \cos x$

$\cos(x+y) = \cos x \cos y - \sin y \sin x.$

---

1) The values of x for which $|5x-3| \le 2$ are

a) $1 \le x \le 2$

b) $1 < x < 5$

c) $x > 0$

d) $\frac{1}{5} \le x \le 1$

e) $1 \le x \le 5$

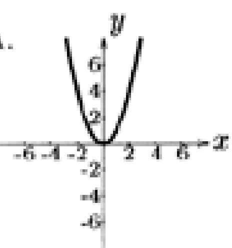
$\rightarrow -2 \le 5x-3 \le 2$

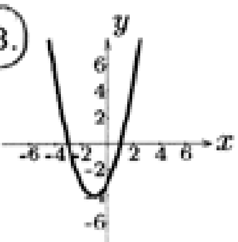
$\rightarrow 1 \le 5x \le 5$

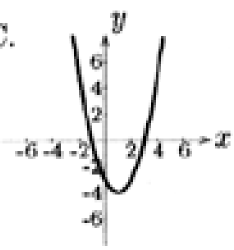
$\rightarrow \frac{1}{5} \le x \le 1$

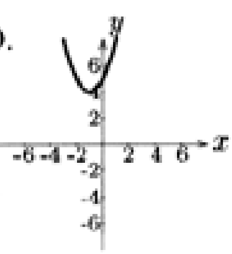
→ answer is d

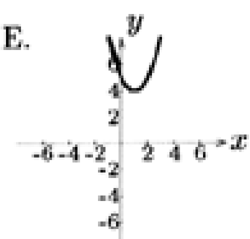
2) The graph of $y - x^2 - 2x + 3 = 0$ looks most like

A. 

B. 

C. 

D. 

E. 

$\rightarrow y - (x^2 + 2x \quad) + 3 = 0$

$\rightarrow y - (x^2 + 2x + 1) + 3 = 0 + 1$

$\rightarrow y - (x+1)^2 + 3 = 1$

$\rightarrow y + 4 = (x+1)^2$

$\rightarrow y - (-4) = (x-(-1))^2$

translate graph of $y = x^2$ 1 unit to left, 4 units down

→ B

3) Let $f(x) = \dfrac{1}{\sqrt{x+1}}$ and $g(x) = \dfrac{1}{x+2}$. Then, for the values of x for which $f \circ g(x)$ or $g \circ f(x)$ is defined, which of the following is correct?

a) $f \circ g(x) = \sqrt{\dfrac{x+2}{x+3}}$

b) $f \circ g(x) = \sqrt{\dfrac{x+3}{x+2}}$

c) $g \circ f(x) = \sqrt{\dfrac{x+3}{x+2}}$

d) $g \circ f(x) = \sqrt{\dfrac{x+2}{x+3}}$

e) $g \circ f(x)$ is not defined for any value of x

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x+2}\right) = \frac{1}{\sqrt{\frac{1}{x+2}+1}} = \frac{1}{\sqrt{\frac{1+(x+2)}{x+2}}} = \frac{1}{\sqrt{\frac{x+3}{x+2}}} = \sqrt{\frac{x+2}{x+3}} \rightarrow$$ answer is a.

$$(g \circ f)(x) = g\left(\frac{1}{\sqrt{x+1}}\right) = \frac{1}{\frac{1}{\sqrt{x+1}}+2} = \frac{1}{\frac{1+2\sqrt{x+1}}{\sqrt{x+1}}} = \frac{\sqrt{x+1}}{1+2\sqrt{x+1}}$$, no choices are like this.

4) What is the domain of the function $f(x) = \ln\left(\dfrac{x-1}{2-x}\right)$?

a) $x > 1$

b) $1 < x < 2$

c) $x \neq 2$

d) $x > 2$

e) $x < 1$ or $x > 2$

$\dfrac{x-1}{2-x} > 0$

x: $-\infty$ … ∞	1		2	
$x-1$	$-$ 0	$+$		$+$
$2-x$	$+$	$+$	0	$-$
$\dfrac{x+1}{2-x}$	$-$	$+$		$-$

$\dfrac{x+1}{2-x} > 0 \rightarrow 1 < x < 2 \rightarrow$ b

5) For what values of x in $[0, 2\pi]$ is $\cos x < \frac{\sqrt{3}}{2}$?

a) $0 < x < \frac{\pi}{6}$

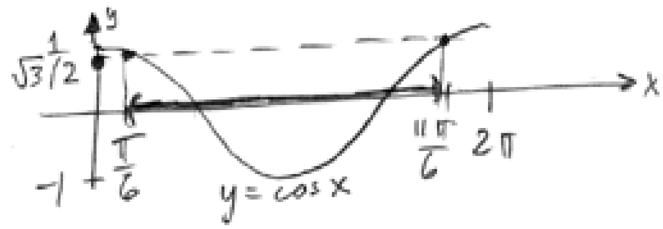
b) $\frac{\pi}{6} < x < \frac{11\pi}{6}$

c) $\frac{\pi}{3} < x < 2\pi$

d) $\frac{\pi}{3} < x < \frac{4\pi}{3}$

e) $0 < x < \frac{\pi}{3}$ and $\frac{2\pi}{3} < x \leq 2\pi$.

$\cos x = \frac{\sqrt{3}}{2} \rightarrow x = \frac{\pi}{6}, \frac{11\pi}{6}$



$\cos x < \frac{\sqrt{3}}{2} \rightarrow \frac{\pi}{6} < x < \frac{11\pi}{6}$

$\rightarrow$ b.

6) Which ones of the following statements are true?

I) $\sin(x + 2\pi) = \sin x$ and $\cos(x + 2\pi) = \cos x$.

II) $\tan(x + \pi) = \tan x$.

III) $\sin\left(\frac{\pi}{2} - x\right) = \cos(x)$.

IV) $\sin\left(\frac{85\pi}{73}\right) = 1.01238731\ldots$

T I. $\sin(x+2\pi) = \sin x \cos 2\pi + \sin 2\pi \cos x$
$= \sin x$

$\cos(x+2\pi) = \cos x \cos 2\pi - \sin 2\pi \sin x$
$= \cos x$

a) I, II and III are correct

b) I, III and IV are correct

c) Only I and III are correct

d) Only I and IV are correct.

e) All four are correct

T II tangent fcn. has period π.
$\rightarrow \tan(x+\pi) = \tan x$

T III $\sin\left(\frac{\pi}{2} - x\right) = \sin\frac{\pi}{2}\cos(-x) + \sin(-x)\cos\frac{\pi}{2}$
$= (1)\cos(-x) + \sin(-x) \cdot 0$
$= \cos(-x)$
$= \cos x$, since cosine is even

F IV $-1 \leq \sin x \leq 1$ for all x.

$\Rightarrow$ a. is correct

7) An equation of the line tangent to $y = x^2 - 1$ at $(1, 0)$ is

a) $y = 3x - 3$

b) $2y = 5x - 5$

c) $y = 2x - 2$

d) $3y = x - 1$

e) $y = x$

slope of tangent line is derivative, $y'(1)$.

$$y'(1) = \lim_{x \to 1} \frac{y(x) - y(1)}{x-1} = \lim_{x \to 1} \frac{(x^2-1) - (0)}{x-1}$$

$$= \lim_{x \to 1} x+1 = 2$$

tangent line: $y - 0 = 2(x-1)$

$\rightarrow y = 2x - 2$

$\rightarrow$ c is correct

8) The value of the limit

$$\lim_{x \to 0} \frac{\sin 2x}{\sin 3x} \quad \text{is}$$

a) $\frac{3}{4}$

b) $\frac{2}{5}$

c) $\frac{2}{3}$

d) $\frac{2}{7}$

e) not defined.

$$\lim_{x \to 0} \frac{\sin 2x}{\sin 3x} = \lim_{x \to 0} \frac{(2x)\frac{\sin 2x}{2x}}{(3x)\frac{\sin 3x}{3x}}$$

$$= \lim_{x \to 0} \left(\frac{2}{3}\right)\left(\frac{\frac{\sin 2x}{2x}}{\frac{\sin 3x}{3x}}\right) = \frac{2}{3} \cdot \frac{1}{1} = \frac{2}{3}$$

$\rightarrow$ c is correct

9) The value of the limit

$$\lim_{x\to 3} \frac{\sqrt{x}-\sqrt{3}}{x^2-3x} \quad \text{is}$$

a) $\frac{1}{6\sqrt{3}}$

b) $\frac{1}{2\sqrt{3}}$

c) $\frac{1}{\sqrt{3}}$

d) $2\sqrt{3}$

e) $\sqrt{3}$

$$\lim_{x\to 3} \frac{\sqrt{x}-\sqrt{3}}{x^2-3x}$$

$$= \lim_{x\to 3} \frac{\sqrt{x}-\sqrt{3}}{x(x-3)}$$

$$= \lim_{x\to 3} \frac{\cancel{\sqrt{x}-\sqrt{3}}^{\,1}}{x(\sqrt{x}+\sqrt{3})\underbrace{\cancel{(\sqrt{x}-\sqrt{3})}}_{1}}$$

$$= \lim_{x\to 3} \frac{1}{x(\sqrt{x}+\sqrt{3})} = \frac{1}{3(2\sqrt{3})} = \frac{1}{6\sqrt{3}}$$

$\rightarrow$ a is correct.

10) The limit

$$\lim_{x\to 4^+} \frac{x+8}{x^2-16} \quad \text{is}$$

a) $-\infty$

b) $-\frac{1}{2}$

c) 0

d) 12

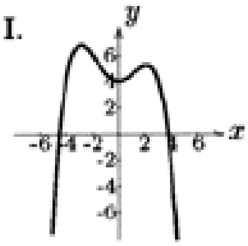
e) ∞

$$\lim_{x\to 4^+} \frac{x+8}{(x-4)(x+4)} = \frac{12}{(0^+)(8)} = \infty$$

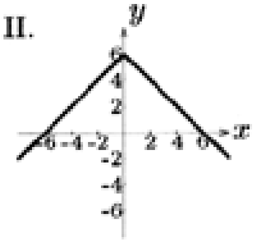
$\rightarrow$ e is correct

11) Which of the following represent the graph of a continuous function at $x = 0$?

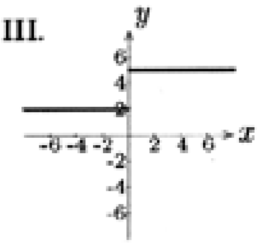
I.



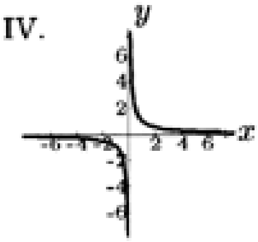
II.



III.



IV.



I cont at X=0

II cont at X=0

III not cont. at X=0 because jump at X=0

IV not cont at X=0. Big jump at X=0

→ e is correct

a) all four

b) I and III

c) II and III

d) III and IV

e) I and II

12) Among the graphs above, which ones represent a function that is differentiable at $x = 0$?

a) I and II

b) I and III

c) II and III

d) Only I

e) all four.

Only I is differentiable at X=0. It's the only one that's smooth at X=0.

→ d is answer.

II. is not differentiable at X=0 because graph has a corner there.