MATH 161 – FALL 1999 – SECOND EXAM
October 28, 1999

STUDENT NAME: SOLUTION KEY

STUDENT ID NUMBER: ______________________

RECITATION INSTRUCTOR: ______________________

INSTRUCTIONS:
1. This test booklet has 6 pages including this one.
2. Fill in your name, your student ID number, and your recitation instructor's name above.
3. Use a number 2 pencil on the mark-sense sheet (answer sheet).
4. On the mark-sense sheet, fill in the recitation instructor's name and the course number.
5. Fill in your name and student ID number, blacken the appropriate spaces, and sign the mark-sense sheet.
6. Mark the the division and section number of your class and blacken the corresponding circles, including the circles for the zeros. If you do not know your division and section number ask your instructor.
7. There are 11 questions, each worth 9 points. Blacken your choice of the correct answer in the spaces provided for questions 1-11. Turn in BOTH the answer sheet and the question sheets to your instructor when you are finished.
8. No books, notes or calculators may be used.

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1) $\frac{d}{dx}\left(\frac{\ln(x^2)}{x}\right) =$

$$\frac{\left(\frac{1}{x^2}\right)(2x)(x) - (\ln x^2)(1)}{x^2} = \frac{2 - \ln x^2}{x^2} = \frac{2 - 2\ln x}{x^2}.$$

a) $\frac{\ln x}{x^2}$

b) $\frac{x^3}{\ln x^3}$

c) $\frac{\ln x^3}{x^3}$

d) $\frac{2 - 2\ln x}{x^2}$

e) $(\ln x)^2$

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**2)** A curve on the plane is given by $3x^2 + y^2 = 7$. Which of the following is the slope of the tangent line to this curve at $(1, 2)$?

a) $-\frac{3}{2}$

b) $-\frac{1}{2}$

c) 1

d) $\frac{3}{2}$

e) $\frac{7}{2}$

Differentiate w.r.t. x $\Rightarrow 6x + 2y\frac{dy}{dx} = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{-6x}{2y} = \frac{-3x}{y}$$

$$\therefore \left.\frac{dy}{dx}\right|_{(1,2)} = \frac{-3(1)}{2} = \frac{-3}{2}$$

**3)** Approximate $(24)^{1/3}$ using linear approximation.

a) $\frac{8}{3}$

b) $\frac{14}{5}$

c) $\frac{26}{9}$

d) $\frac{27}{10}$

e) 3

Let $f(x) = x^{1/3}$. Note: $(24)^{1/3} = (27-3)^{1/3}$

$f'(x) = \frac{1}{3}x^{-2/3}$.

$$(27-3)^{1/3} \approx f(27) + f'(27)\,dx$$

$$= 27^{1/3} + \frac{1}{3}(27)^{-2/3}(-3)$$

$$= 3 + \frac{1}{3}\left(\frac{1}{9}\right)(-3)$$

$$= 3 - \frac{1}{9}$$

$$= \frac{26}{9}.$$

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**4)** The maximum and minimum of $f(x) = x^2 - 2x^3$ on $[0, 1]$ are

a) maximum $\frac{1}{3}$, minimum 0

b) maximum $\frac{1}{9}$, minimum 0

c) maximum $\frac{1}{27}$, minimum 0

d) maximum $\frac{1}{27}$, minimum $-1$

e) maximum $\frac{1}{9}$, minimum $-1$

Find critical points:

$$f'(x) = 2x - 6x^2$$
$$= 2x(1-3x)$$
$$= 0 \rightarrow x = 0, \tfrac{1}{3}$$

| $x$ | $f(x) = x^2 - 2x^3$ | |
|---|---|---|
| $0$ | $0 - 0 = 0$ | |
| $\frac{1}{3}$ | $\frac{1}{9} - \frac{2}{27} = \frac{1}{27}$ | maximum |
| $1$ | $1 - 2 = -1$ | minimum |

**5)** The interval(s) on which the function $f(x) = x^3 - 3x^2 - 9x + 1$ is increasing is (are)

a) $[-1, 3]$

b) $(-\infty, 3]$

c) $(-\infty, -1]$ and $[0, 3]$

d) $[-1, 0]$ and $[3, \infty)$

e) $(-\infty, -1]$ and $[3, \infty)$

$$f'(x) = 3x^2 - 6x - 9$$
$$= 3(x^2 - 2x - 3)$$
$$= 3(x-3)(x+1)$$
$$= 0 \rightarrow x = -1, 3$$

| | $-\infty$ to $-1$ | $-1$ to $3$ | $3$ to $\infty$ |
|---|---|---|---|
| $x+1$ | $-$ | $+$ | $+$ |
| $x-3$ | $-$ | $-$ | $+$ |
| $f'(x)$ | $+$ | $-$ | $+$ |
| $f$ | inc. | dec. | inc. |

**6)** An exponentially growing population of certain bacteria doubles in size every three days. How many days will it take for the population to be eight times the intial one?

a) $\frac{\ln 8}{\ln 3}$

b) $3 \ln 8$

c) 9

d) 12

e) 10

Method 1:

| $t$ | $P(t)$ |
|---|---|
| 0 | $P(0)$ |
| 3 | $2P(0)$ |
| 6 | $2(2P(0)) = 4P(0)$ |
| 9 | $2(4P(0)) = 8P(0)$ |

Method 2: $2P(0) = P(0)e^{3k} \to 2 = e^{3k} \to \ln 2 = 3k \to k = \frac{1}{3}\ln 2$

$P(t) = P(0)e^{(\frac{1}{3}\ln 2)t}$

Solve for $t$: $8P(0) = P(0)e^{(1/3 \ln 2)t} \to 8 = e^{(\frac{1}{3}\ln 2)t} \to \ln 8 = (\frac{1}{3}\ln 2)$

$\to t = 3\frac{\ln 8}{\ln 2} = 3\frac{\ln 2^3}{\ln 2} = 3\frac{3\ln 2}{\ln 2} = 3 \cdot 3 = 9.$

**7)** The derivative of a function $f(x)$ is $f'(x) = (x-1)^2(x-2)(x+3)$. Which of the following is correct?

a) $f(1)$ is a relative maximum, $f(2)$ and $f(-3)$ are relative minima.

b) $f(1)$ is a relative minimum, $f(2)$ and $f(-3)$ are relative maxima.

c) $f(2)$ is a relative minimum, $f(1)$ and $f(-3)$ are relative maxima.

d) $f(-3)$ is a relative maximum, $f(2)$ a relative minimum and $f(1)$ is neither.

e) $f(2)$ is a relative maximum, $f(-3)$ a relative minimum and $f(1)$ is neither.

| | $-\infty$ to $-3$ | $-3$ to $1$ | $1$ to $2$ | $2$ to $\infty$ |
|---|---|---|---|---|
| $x+3$ | $-$ | $+$ | $+$ | $+$ |
| $(x-1)^2$ | $+$ | $+$ | $+$ | $+$ |
| $x-2$ | $-$ | $-$ | $-$ | $+$ |
| $f'(x)$ | $+$ | $-$ | $-$ | $+$ |

$f$

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**8)** A 10 ft tall lamp post casts a shadow on level ground of a 6 ft person walking away from the lamp post. Find the rate at which the shadow is increasing when the person is walking at 3 miles per hour.

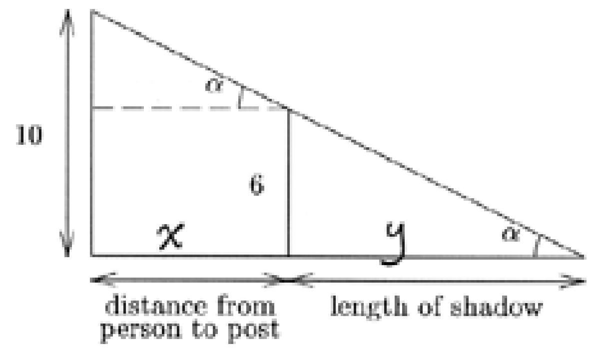
a) $\frac{5}{3}$ miles per hour

b) 2 miles per hour

c) 6 miles per hour

d) $\frac{3}{2}$ miles per hour

e) $\frac{9}{2}$ miles per hour




Let $x$ = distance of person to lamp post.

Let $y$ = length of shadow.

Know: $dx/dt = 3$. Want: $dy/dt$.

By similar triangles: $\frac{x+y}{10} = \frac{y}{6} \rightarrow 6x + 6y = 10y$

$\rightarrow 6x = 4y \rightarrow 6\frac{dx}{dt} = 4\frac{dy}{dt} \rightarrow \frac{dy}{dt} = \frac{6}{4}\frac{dx}{dt} = \frac{3}{2}(3) = \frac{9}{2}$.

**9)** The product of three positive numbers is 48, one of them is three times another. Find the minimum of their sum.

a) 6

b) 12

c) 16

d) 24

e) 48

Let $x > 0$, $y > 0$, $z > 0$. $xyz = 48$.

Let $z = 3y$. Then $xy(3y) = 3xy^2 = 48$

$\rightarrow x = \frac{48}{3y^2} = \frac{16}{y^2}$

Find minimum of $S = x + y + z = \frac{16}{y^2} + y + 3y$

$= 16y^{-2} + 4y$.

$\frac{dS}{dy} = -32y^{-3} + 4 = \frac{-32 + 4y^3}{y^3} = 0 \rightarrow y = 2$.

sign of $S'$: 0, 2 → $y$

$4(y^3 - 8)$: $-$ 0 $+$

$S$: min.

Sum: $S = 16(2)^{-2} + 4(2)$

$= 16(\frac{1}{4}) + 4(2)$

$= 4 + 8 = 12$

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**10)** What is the value of the following limit? $\lim_{x\to\infty}\left(4+3x-\sqrt{9x^2+4}\right) = \infty - \infty$

a) 0

b) 4

c) $\infty$

c) 1

e) 2

Method 1: Note: $\sqrt{9x^2+4} \approx 3x$ for large x.

$\therefore \lim_{x\to\infty}\left(3x-\sqrt{9x^2+4}\right) = 0$

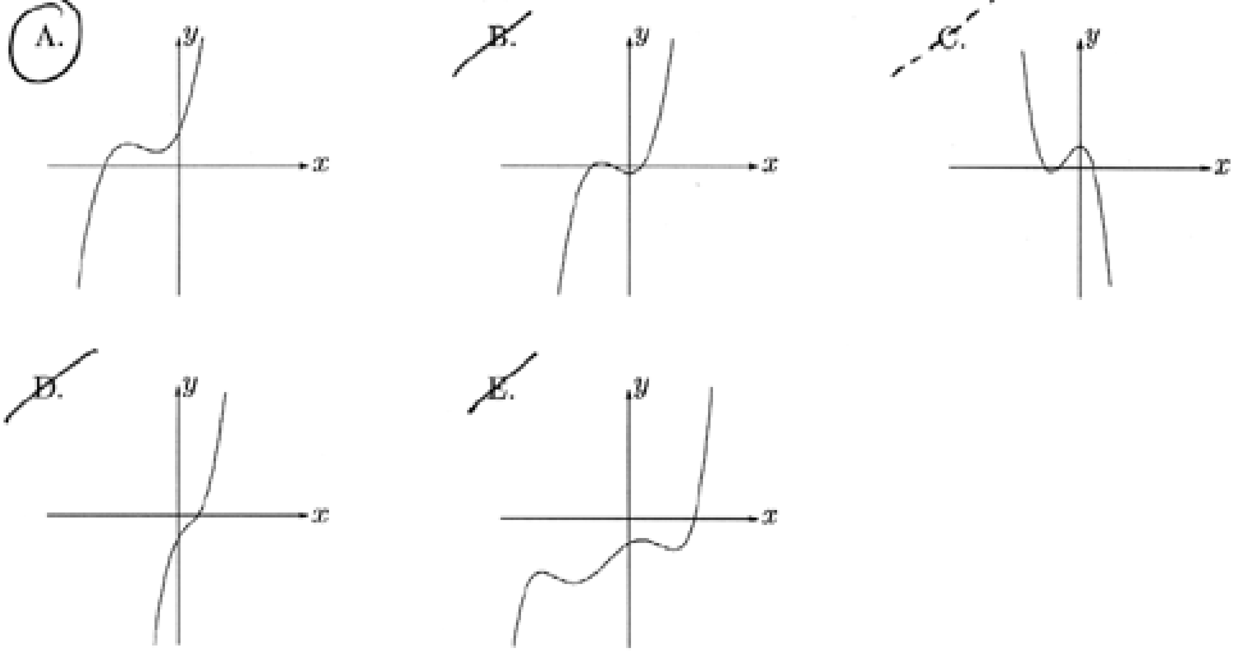
Thus, $\lim_{x\to\infty}\left(4+3x-\sqrt{9x^2+4}\right) = 4+0 = 4$

Method 2: $\left((4+3x)-\sqrt{9x^2+4}\right)\left(\frac{(4+3x)+\sqrt{9x^2+4}}{4+3x+\sqrt{9x^2+4}}\right)$

$= \frac{16+24x+9x^2-(9x^2+4)}{4+3x+\sqrt{9x^2+4}} = \frac{24x+12}{4+3x+\sqrt{9x^2+4}}$

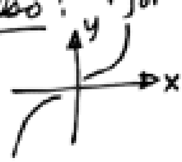
and $\lim_{x\to\infty}\frac{24x+12}{4+3x+\sqrt{9x^2+4}} = \lim_{x\to\infty}\frac{24x+12}{4+3x+3x} = \frac{24}{3+3} = 4.$

**11)** The graph of the function $f(x)=x^3+2x^2+x+1$ looks most like




Note: $f(0) = 0+0+0+1 = 1$. Thus choices B, D and E are eliminated.

Note also: for large x, $f(x) \approx x^3$ and $y=x^3$ has graph




so C is eliminated. $\Rightarrow$ A is answer.