MATH 161 – FALL 1999 – THIRD EXAM
November 30, 1999

STUDENT NAME: SOLUTION KEY

STUDENT ID NUMBER: ____________________

RECITATION INSTRUCTOR: ____________________

INSTRUCTIONS:
1. This test booklet has 6 pages including this one.
2. Fill in your name, your student ID number, and your recitation instructor's name above.
3. Use a number 2 pencil on the mark-sense sheet (answer sheet).
4. On the mark-sense sheet, fill in the recitation instructor's name and the course number.
5. Fill in your name and student ID number, blacken the appropriate spaces, and sign the mark-sense sheet.
6. Mark the the division and section number of your class and blacken the corresponding circles, including the circles for the zeros. If you do not know your division and section number ask your instructor.
7. There are 11 questions, each worth 9 points. Blacken your choice of the correct answer in the spaces provided for questions 1-11. Turn in BOTH the answer sheet and the question sheets to your instructor when you are finished.
8. No books, notes or calculators may be used.

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1) Let $P = \{-1, -\frac{1}{2}, 0, \frac{1}{2}, 1\}$ be a partition of $[-1, 1]$. The approximation of

$$\int_{-1}^{1} (x^2 + 1)\, dx$$

given by the right sum with respect to P is

a) $\frac{11}{4}$

b) 1

c) 2

d) $\frac{3}{2}$

e) 3

$-1 \quad -\frac{1}{2} \quad 0 \quad \frac{1}{2} \quad 1$ $\quad x$

$x^2+1 \qquad \frac{5}{4} \quad 1 \quad \frac{5}{4} \quad 2$

$$\sum (x^2+1)\left(\tfrac{1}{2}\right) = \left(\tfrac{5}{4}\right)\left(\tfrac{1}{2}\right) + (1)\left(\tfrac{1}{2}\right) + \left(\tfrac{5}{4}\right)\left(\tfrac{1}{2}\right) + (2)\left(\tfrac{1}{2}\right)$$

$$= \left(\tfrac{5}{4} + 1 + \tfrac{5}{4} + 2\right)\left(\tfrac{1}{2}\right) = \left(\tfrac{22}{4}\right)\left(\tfrac{1}{2}\right) = \tfrac{11}{4}$$

2) Let f be a continuous function in $[0,4]$ satisgying $f(4)=3$. Let h be a differentiable function satisfying $h(1)=4$ and $h'(1)=2$. Then

$$\frac{d}{dx}\int_0^{h(x)} f(t)\,dt, \text{at } x=1$$

is equal to

a) 6

b) 2

c) 3

d) 1

e) 0

$$\frac{d}{dx}\int_0^{h(x)} f(t)dt = f(h(x)) \cdot h'(x)$$

$$x=1 \longrightarrow f(h(1)) \cdot h'(1)$$

$$= f(4) \cdot 2$$

$$= 3 \cdot 2$$

$$= 6$$

3)

$$\int (x^2+x)\left(\frac{1}{x}+x\right)\,dx =$$

a) $\left(\frac{x^3}{3}+\frac{x^2}{2}\right)\left(\ln|x|+\frac{x^2}{2}\right)+C$

b) $\frac{1}{2}(x^2+x)^2+C$

c) $(2x+1)\left(\frac{-1}{x^2}+1\right)+C$

d) $\frac{x^4}{4}+\frac{x^3}{3}+\frac{x^2}{2}+x+C$

e) $3x^2+2x+1+C$

$$= \int (x + x^3 + 1 + x^2)\,dx$$

$$= \frac{x^2}{2} + \frac{x^4}{4} + x + \frac{x^3}{3} + C$$

4)

$$\int_1^4 \frac{3x+1}{\sqrt{x}}\,dx =$$

a) 0

b) $\frac{7}{2}$

c) 8

d) $\frac{21}{2}$

e) 16

$$= \int_1^4 \left(3x^{1/2} + x^{-1/2}\right) dx$$

$$= \left(3 \cdot \frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2}\right)\Bigg|_1^4$$

$$= \left(2 \cdot 4^{3/2} + 2 \cdot 4^{1/2}\right) - \left(2 \cdot 1^{3/2} + 2 \cdot 1^{1/2}\right)$$

$$= (16 + 4) - (2 + 2)$$

$$= 16$$

5)

$$\int_0^1 \frac{x}{\sqrt{3x^2+1}}\,dx =$$

a) $\frac{1}{2}$

b) $\frac{1}{3}$

c) 3

d) $\frac{1}{4}$

e) 1

$$= \int_0^1 (3x^2+1)^{-1/2}\, x\,dx$$

[Let $u = 3x^2+1$, then $du = 6x\,dx$]

$$= \int_1^4 u^{-1/2}\,\frac{1}{6}\,du$$

$$= \frac{1}{6} \cdot \frac{u^{1/2}}{1/2}\Bigg|_1^4 = \frac{1}{3}\left(4^{1/2} - 1^{1/2}\right) = \frac{1}{3}$$

6)

$$\int_1^{e^2} \frac{2\ln(x)}{x}\,dx =$$

a) 2

b) $2e^2$

c) 3

d) $2\ln 2$

e) 4

[let $u = \ln x$, then $du = \frac{1}{x}\,dx$]

$$= \int_0^2 2u\,du$$

$$= u^2\Big|_0^2 = 2^2 - 0^2 = 4$$

7) The substitution $u = 1 + \sin x$ in the integral

$$\int_0^{\pi/6} (\cos x)(1+\sin x)^{33}\,dx$$

transforms it into

a) $\int_0^{3/2} (\sin u)^{33}\,du$

b) $\int_0^{3/2} u^{33}\,du$

c) $\int_1^{3/2} u^{33}\,du$

d) $\int_0^{3/2} \cos u\,du$

e) $\int_1^{1/2} \cos u(\sin u)^{33}\,du$

$u = 1 + \sin x$

$\rightarrow du = \cos x\,dx$

$u(\pi/6) = 1 + \sin \pi/6 = 1 + \frac{1}{2} = \frac{3}{2}$

$u(0) = 1 + \sin 0 = 1 + 0 = 1$

$\therefore \int_1^{3/2} u^{33}\,du$

8) The area of the region enclosed by the curves $y = x^2$ and $x = 6 - y$ is given by the integral

a) $\displaystyle\int_{-3}^{2} (x^2 - x + 6)\, dx$

b) $\displaystyle\int_{-2}^{2} (6 - y - \sqrt{y})\, dy$

c) $\displaystyle\int_{-3}^{2} (6 - x - x^2)\, dx$

d) $\displaystyle\int_{-2}^{2} (\sqrt{y} - 6 - y)\, dy$

e) $\displaystyle\int_{-2}^{2} (x^2 + x - 6)\, dx$

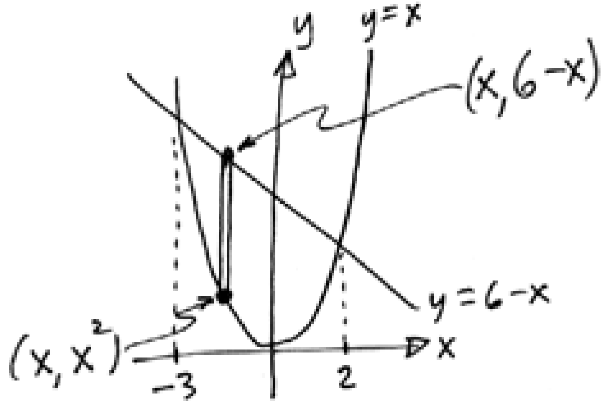
intersection of curves: substitute x^2 for y into $x = 6 - y$

$\rightarrow x = 6 - x^2$

$\rightarrow x^2 + x - 6 = 0$

$\rightarrow (x+3)(x-2) = 0$

$\rightarrow x = 2, -3$



$$\text{Area} = \int_{-3}^{2} ((6-x)-(x^2))\, dx$$

9) The derivative of $f(x) = (1 + x)^x$ is

a) $x(1 + x)^{x-1}$

b) $(1 + x)^x \ln(1 + x)$

c) $(1 + x)^x \left(\dfrac{x}{x+1} + \ln(1 + x)\right)$

d) $(1 + x)x^x$

e) $x\left((1 + x)^x + \ln(1 + x)\right)$

Let $y = (1+x)^x$

Then $\ln y = \ln(1+x)^x$

$\rightarrow \ln y = x \ln(1+x)$

$\rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = (1)(\ln(1+x)) + x\left(\dfrac{1}{1+x}\right)$

$$\rightarrow \frac{dy}{dx} = y\left(\ln(1+x) + \frac{x}{1+x}\right)$$

$$= (1+x)^x\left(\ln(1+x) + \frac{x}{1+x}\right)$$

10) Let $f(x) = x^3 + 4$. Then $(f^{-1})'(5) =$

a) $\frac{1}{2}$

b) $\frac{1}{3}$

c) $\frac{1}{4}$

d) $\frac{1}{5}$

e) 1

Note that $f(1) = 1^3 + 4 = 5$

Now, $(f^{-1})'(5) = \dfrac{1}{f'(1)}$.

$f'(x) = 3x^2 \rightarrow f'(1) = 3$

$\therefore (f^{-1})'(5) = \frac{1}{3}$.

11)

$$\int_0^2 3^x \, dx =$$

a) 18

b) $\frac{18}{\ln 3}$

c) $8 \ln 3$

d) $\frac{8}{\ln 3}$

e) 6

$$= (3^x)\left(\frac{1}{\ln 3}\right)\Big|_0^2$$

$$= 3^2\left(\frac{1}{\ln 3}\right) - 3^0\left(\frac{1}{\ln 3}\right)$$

$$= \frac{1}{\ln 3}(9-1)$$

$$= \frac{8}{\ln 3}$$