1. If $f(x) = \dfrac{2x-1}{x-1}$, then $f'(x) =$

$$f'(x) = \frac{(2)(x-1) - (2x-1)(1)}{(x-1)^2}$$

$$= \frac{2x - 2 - 2x + 1}{(x-1)^2}$$

$$= \frac{-1}{(x-1)^2}$$

A. $\dfrac{4x-3}{(x-1)^2}$

B. $\dfrac{-1}{(x-1)^2}$

C. $\dfrac{-x-1}{(x-1)^2}$

D. $\dfrac{3x-2}{(x-1)^2}$

E. $\dfrac{4x-2}{x-1}$

2. If $f(x) = \ln(\ln x)$, then $f'(e) =$

$$f'(x) = \frac{1}{\ln x} \cdot \frac{1}{x}$$

$$f'(e) = \frac{1}{\ln e} \cdot \frac{1}{e} = \frac{1}{e}$$

A. -1

B. 0

C. $\frac{1}{e}$

D. 1

E. e

3. If $f(x) = e^{x^2} \cos 3x$, then $f''(0) =$

A. 11

B. 2

C. 0

D. -1

E. -7

$f'(x) = e^{x^2}(2x)\cos 3x + e^{x^2}(-\sin 3x)(3)$

$f''(x) = \left[e^{x^2}(2x)(2x)(\cos 3x) + e^{x^2}(2)(\cos 3x) + e^{x^2}(2x)(-\sin 3x)(3) \right] + \left[e^{x^2}(2x)(-\sin 3x)(3) + e^{x^2}(-\cos 3x)(3)(3) \right]$

$f''(0) = [0 + 2 + 0] + [0 + (-9)] = -7$

4. The slope of the line tangent to $x^2 + x^2y^2 + y^3 = 3$ at $(1,1)$ is

A. $-\frac{4}{5}$

B. $-\frac{3}{5}$

C. $-\frac{2}{5}$

D. $-\frac{1}{5}$

E. 0

Differentiate with respect to x $\Rightarrow$

$2x + 2xy^2 + x^2\left(2y\frac{dy}{dx}\right) + 3y^2\frac{dy}{dx} = 0$

$(x,y) = (1,1) \Rightarrow$

$2 + 2 + 2\frac{dy}{dx} + 3\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{4}{5}$

5. A spherical balloon is losing air at the rate of 2 ft^3/min. How fast is the radius of the balloon shrinking when the radius is 4 ft?

A. $\frac{1}{32\pi}$ ft/min

B. $\frac{1}{2\pi}$ ft/min

C. 2π ft/min

D. 32π ft/min

E. $\frac{3}{2\pi}$ ft/min

Know: $\frac{dV}{dt} = -2$ Want: $\frac{dr}{dt}$ at time when $r=4$

$V = \frac{4}{3}\pi r^3 \longrightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\longrightarrow -2 = 4\pi 4^2 \frac{dr}{dt}$

$\longrightarrow \frac{dr}{dt} = \frac{-2}{64\pi} = \frac{-1}{32\pi}$.

6. Using a linear approximation to

$y = x^{\frac{4}{3}}$ at $x = 8$, $(7.5)^{\frac{4}{3}} \approx$

A. $15\frac{2}{3}$

B. $15\frac{1}{3}$

C. 15

D. $14\frac{2}{3}$

E. $14\frac{1}{3}$

Tangent line to $y = x^{4/3}$ has slope $y' = \frac{4}{3}(x^{1/3})$. $y'(8) = \frac{4}{3} \cdot 8^{1/3} = \frac{8}{3}$

$y(8) = 8^{4/3} = 16$

Tangent Line: $y - 16 = \frac{8}{3}(x-8)$

$\longrightarrow y = 16 + \frac{8}{3}(x-8)$

$x = 7.5 \longrightarrow y = 16 + \frac{8}{3}(7.5-8)$

$= 16 + \frac{8}{3}\left(-\frac{1}{2}\right)$

$= 16 - \frac{4}{3} = 14\frac{2}{3}$.

7. $f(x) = 2x^3 + 3x^2 - 12x$ on the interval $[0, 2]$ has

f continuous on closed interval $[0,2]$.

$f'(x) = 6x^2 + 6x - 12 = 6(x+2)(x-1)$.

Critical numbers in $[0,2]$ are $x = 1$.

x	$2x^3 + 3x^2 - 12x$	
0	0	
1	$2+3-12 = -7$	min
2	$16+12-24 = 4$	max

A. maximum value of 20, minimum value of 0

B. maximum value of 20, minimum value of -7

C. maximum value of 4 minimum value of 0

D. maximum value of 8 minimum value of -4

E. maximum value of 4 minimm value of -7

8. For a certain function f with $f'(x) = -2(3x+1)(x-2)$, the interval(s) on which $f(x)$ is increasing is (are)

	$-\infty$ to $-\frac{1}{3}$	$-\frac{1}{3}$ to 2	2 to ∞
-2	$-$	$-$	$-$
$3x+1$	$-$	$+$	$+$
$x-2$	$-$	$-$	$+$
$f'(x)$	$-$	$+$	$-$
f	dec.	incr.	dec.

A. $x < 2$

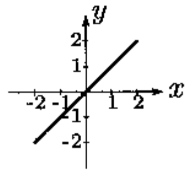
B. $x > -\frac{1}{3}$

C. $-\frac{1}{3} < x < 2$

D. $x < -\frac{1}{3}$ or $x > 2$

E. $x < -\frac{1}{3}$

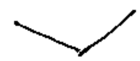
9. The following is a graph of f' for $-2 \le x \le 2$



$\Rightarrow$ $-2 \quad 0 \quad 2$

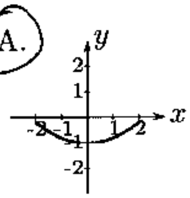
$f'(x) \quad - \quad 0 \quad +$

$\Rightarrow$ graph of f

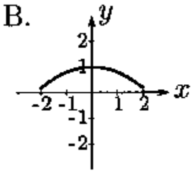


which of the following could be a graph of f?

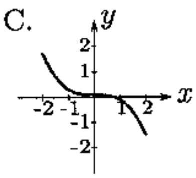
A.



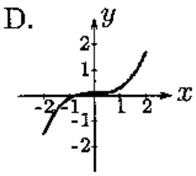
B.



C.



D.



E. More information is needed to determine the graph of f.

10. Let $f'(x) = x^2 + x - 2$. First find $f(x)$ so that $f(1) = 0$. Then $f(2)$ is

$\rightarrow f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + C$

$f(1) = 0 = \frac{1}{3} + \frac{1}{2} - 2 + C \rightarrow C = \frac{7}{6}$

$\rightarrow f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + \frac{7}{6}$

$\rightarrow f(2) = \frac{8}{3} + 2 - 4 + \frac{7}{6} = \frac{11}{6}$

A. $\frac{7}{6}$

B. 4

C. 0

D. $-\frac{1}{6}$

E. $\frac{11}{6}$

11. A population is growing exponentially. It was 250 twenty four years ago and 500 eight years ago. How large is it now? ($\sqrt{2} \approx 1.414$)

(A.) 707

B. $500e^{0.8}$

C. 750

D. $120e^{24}$

E. 1359

$P(t) = P(0)e^{kt}$.

Let: $P(0) = 250$, $P(16) = 500$.

Want: $P(24) = ?$

$P(t) = 250e^{kt}$, $P(24) = 250e^{24k}$

need k: $P(16) = 500 = 250e^{16k}$

$\rightarrow 2 = e^{16k} \rightarrow \ln 2 = 16k \rightarrow k = \frac{1}{16}\ln 2$

$\therefore P(24) = 250e^{(\frac{1}{16}\ln 2)(24)} = 250\left(e^{\ln 2}\right)^{\frac{24}{16}} = 250(2)^{3/2}$

$= 250 \cdot 2 \cdot 2^{1/2} = 500\sqrt{2} = (500)(1.414) \cong 707$

12. The function $f(x) = 4x^2 - \frac{1}{x}$ has

A. a relative maximum at $x = \frac{1}{2}$

(B.) a relative minimum at $x = -\frac{1}{2}$

C. a relative maximum at $x = \frac{1}{2}$

D. a relative minimum at $x = \frac{1}{2}$

E. No extreme values

$f'(x) = 8x + \frac{1}{x^2}$

$= \frac{8x^3 + 1}{x^2} = 0$

$\longrightarrow x = -\frac{1}{2}$

$-1/2$ x

$8x^3+1$ $-$ 0 $+$

f

min at $x = -\frac{1}{2}$

13. Given the following graph of $f'(x)$ we see that $f(x)$ has

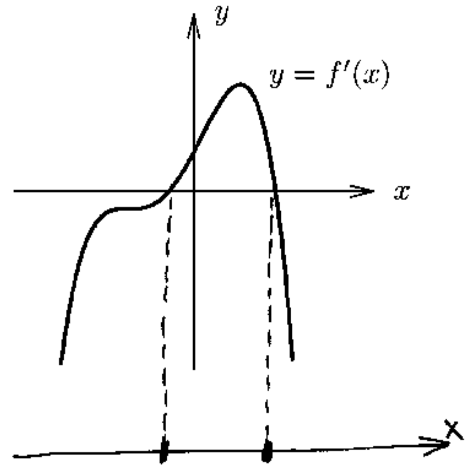
A. one relative maximum and no relative minimum

B. no relative maximum and one relative minimum

C. one relative maximum and one relative minimum

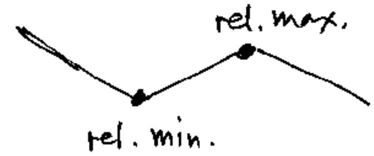
D. no relative maximum and two relative minima

E. one relative maximum and two relative minima



$f'(x)$ $-$ 0 $+$ 0 $-$

graph of f



14. The concentration of a drug in the blood stream t seconds after injection into a muscle is given by

$y = 14(e^{-0.01t} - e^{-0.01et}), \quad t \geq 0. \qquad (e \approx 2.718)$

Then the concentration is largest after

A. 58 sec

B. 14 sec

C. 1400 sec

D. 272 sec

E. 117 sec

$$\frac{dy}{dx} = 14\left(-0.01\, e^{-0.01t} + (0.01e)e^{-0.01et}\right)$$

$$= 14\left(-0.01\, e^{-0.01t} + 0.01\, e^{1-0.01et}\right)$$

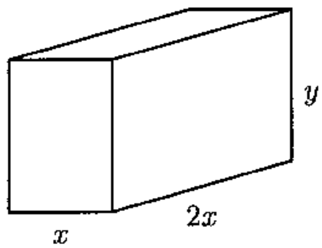
$$\frac{dy}{dx} = 0 \rightarrow \cancel{0.01}\, e^{-0.01t} = \cancel{0.01}\, e^{1-0.01et}$$

$$\rightarrow -0.01t = 1 - 0.01et$$

$$\rightarrow 0.01et - 0.01t = 1$$

$$\rightarrow t = \frac{1}{0.01(e-1)} = \frac{100}{e-1} \cong \frac{100}{1.718} \approx 58$$

15. A crate has 4 rectangular sides, rectangular top and bottom, twice as long as they are wide, and a volume V. If the crate has the smallest possible surface area, the width of the base is



A. $\sqrt[3]{\dfrac{3V}{2}}$

B. $\dfrac{\sqrt[3]{3V}}{2}$

C. $\sqrt[3]{V}$

D. $\dfrac{\sqrt[3]{V}}{2}$

E. $\dfrac{\sqrt[3]{V}}{3}$

volume: $V = 2x^2 y \longrightarrow y = \dfrac{V}{2x^2}$

Surface Area:
$$S = 2(xy) + 2(2xy) + 2(2x^2)$$
$$= 6xy + 4x^2$$
$$= 6x\left(\frac{V}{2x^2}\right) + 4x^2$$
$$= \frac{3V}{x} + 4x^2$$

$$\frac{dS}{dx} = \frac{-3V}{x^2} + 8x = \frac{-3V + 8x^3}{x^2} = 0 \rightarrow x = \sqrt[3]{\frac{3V}{8}}$$
$$= \frac{\sqrt[3]{3V}}{2}$$

$\sqrt[3]{\frac{3V}{8}}$

x

$\dfrac{dS}{dx}$ $-$ 0 $+$

S

min.