MATH 161 – 161E – SPRING 1998 – THIRD EXAM – April 16, 1998

STUDENT NAME: SOLUTION KEY

STUDENT ID: ______________________

RECITATION INSTRUCTOR: ______________________

INSTRUCTIONS:

1. This test booklet has 4 pages including this one.
2. Fill in your name, your student ID number, and your recitation instructor's name above.
3. Use a number 2 pencil on the mark-sense sheet (answer sheet).
4. On the mark-sense sheet, fill in the recitation instructor's name and the course number.
5. Fill in your name and student ID number, blacken the appropriate spaces, and sign the mark-sense sheet.
6. Mark the division and section number of your class and blacken the corresponding circles, including the circles for the zeros. If you do not know your division and section number ask your instructor.
7. There are 10 questions, each worth 10 points. Blacken your choice of the correct answer in the spaces provided for questions 1-10. Turn in BOTH the answer sheet and the question sheets to your instructor when you are finished.
8. No books, notes or calculators may be used.

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1) The graph of the function $f(x) = 3x^5 - 5x^4$ is concave upward for

a) $x = 0$

b) $x > \frac{4}{3}$

c) $x < 1$

d) $x > 1$

e) $x < \frac{4}{3}$

$f'(x) = 15x^4 - 20x^3$

$f''(x) = 60x^3 - 60x^2 = 60x^2(x-1)$

x		0		1	
x^2	+	0	+		+
$x-1$	−		−	0	+
$f''(x)$	−	0	−	0	+

2) According to a model, the rate R at which a tumor grows is given by

$$R(x) = Ax\ln\left(\frac{B}{x}\right), \quad \text{for } 0 < x < B,$$

where A and B are two positive constants and x is the radius of the tumor. The radius for which the tumor is growing most rapidly is

a) impossible to determine

b) $\frac{A}{B}$

c) $\frac{B}{e}$

d) $\frac{A}{e}$

e) 2

$$R'(x) = A\ln\left(\frac{B}{x}\right) + Ax\left(\frac{x}{B}\right)\left(\frac{-B}{x^2}\right) = A\ln\frac{B}{x} - A$$

$$= 0 \to A\left(\ln\frac{B}{x} - 1\right) = 0 \to \ln\frac{B}{x} = 1$$

$$\to \frac{B}{x} = e$$

$$\to x = \frac{B}{e}$$

0 B/e x

note: $\ln\frac{B}{x}$ + 0 −

3) A crate has vertical sides, square top and bottom, and a volume of 4 cubic meters. If its surface area is as small as possible, then the length of each side of the base (in meters) is.

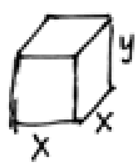
a) 3

b) 12

c) 2

d) $2^{1/2}$

e) $2^{2/3}$



$$x^2y = 4, \quad A = 2x^2 + 4xy$$

$$y = \frac{4}{x^2} \to A = 2x^2 + 4x\left(\frac{4}{x^2}\right) = 2x^2 + \frac{16}{x}$$

$$\frac{dA}{dx} = 4x - \frac{16}{x^2} = \frac{4x^3-16}{x^2} = 0 \Rightarrow x = 4^{1/3} = (2^2)^{1/3}$$

$2^{2/3}$ x

$4(x^3-4)$ − 0 +

4) $\lim_{x\to\infty} \frac{x^3 - x + 1}{3x^2 - 2x + 4x^3} =$

a) $\frac{-1}{2}$

b) 0

c) $\frac{1}{4}$

d) $\frac{1}{3}$

e) ∞

$$= \lim_{x\to\infty} \frac{1 - \frac{1}{x^2} + \frac{1}{x^3}}{\frac{3}{x} - \frac{2}{x^2} + 4} = \frac{1}{4}$$

5) The horizontal asymptote of $y = x - \sqrt{x^2 - 3x}$ is

a) $y = 0$

b) $y = \frac{1}{2}$

c) $y = 1$

d) $y = \frac{3}{2}$

e) $y = 2$

$$\left(x - \sqrt{x^2-3x}\right)\left(\frac{x+\sqrt{x^2-3x}}{x+\sqrt{x^2-3x}}\right) = \frac{x^2 - x^2 + 3x}{x + \sqrt{x^2-3x}}$$

$$\lim_{x\to\infty} \frac{3x}{x+\sqrt{x^2-3x}} = \lim_{x\to\infty} \frac{3}{1+\sqrt{1-\frac{3}{x}}} = \frac{3}{2}$$

6) The area of the region above the line $y = 1$ and below the curve $y = 2 - x^2$ is equal to

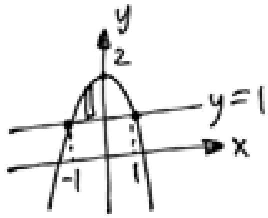
a) $\frac{2}{3}$

b) $\frac{4}{3}$

c) 1

d) $\frac{3}{2}$

e) 2



intersection: $1 = 2 - x^2 \rightarrow x^2 = 1 \rightarrow x = \pm 1$

$$\text{area} = \int_{-1}^{1} \left(\left(2-x^2\right) - (1)\right) dx$$

$$= \int_{-1}^{1} (1 - x^2)\, dx = \left(x - \frac{x^3}{3}\right)\Big|_{-1}^{1} = \left(\frac{2}{3}\right) - \left(-\frac{2}{3}\right) = \frac{4}{3}$$

7) If $F(x) = \int_1^{x^3} \frac{1}{t} dt$, then $F'(1) =$

a) 3

b) $\frac{1}{2}$

c) 1

d) 0

e) 2

$$F'(x) = \frac{1}{x^3}\left(3x^2\right) = \frac{3}{x}$$

$$F'(1) = 3$$

8) $\int_0^1 \frac{x^3}{\sqrt{1+x^4}} dx =$

a) 2

b) $\frac{1}{2}\left(\sqrt{2} - 1\right)$

c) π

d) $\sqrt{2}$

e) 0

Let $u = 1 + x^4$. Then $du = 4x^3\, dx$

$\left(x^3 dx = \frac{1}{4} du\right)$

Substituting, $\int_1^2 u^{-1/2} \frac{1}{4} du$

$$= \frac{1}{4}\, 2u^{1/2}\Big|_1^2 = \frac{1}{2}\left(2^{1/2} - 1\right)$$

9) $\int_{\pi/4}^{\pi/2} \cot(x)dx =$

a) $\frac{\ln 2}{2}$

b) $\frac{1}{2}$

c) 1

d) 0

e) $\frac{\pi}{3}$

$$\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin x}\,dx = \ln(\sin x)\Big|_{\pi/4}^{\pi/2}$$

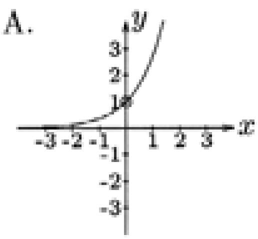
$$= \ln\left(\sin\frac{\pi}{2}\right) - \ln\left(\sin\frac{\pi}{4}\right)$$

$$= \ln 1 - \ln\left(\frac{1}{\sqrt{2}}\right)$$

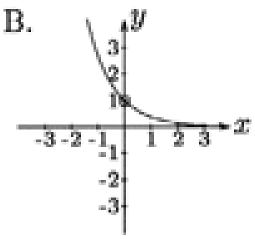
$$= -\ln 2^{-1/2} = \frac{1}{2}\ln 2$$

10) The graph of $y = e^{-1/x}$ looks most like

A.



B.



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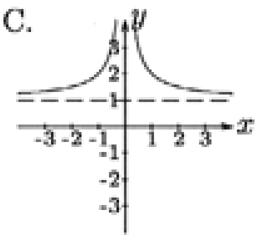

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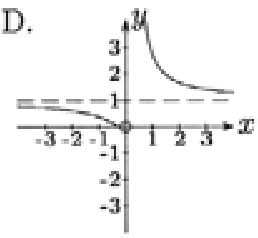
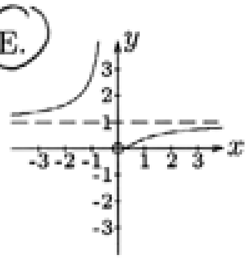


E.



$$\lim_{\substack{x\to\infty \\ x\to-\infty}} e^{-\frac{1}{x}} = e^{0} = 1 \Rightarrow \text{cannot be A or B}$$

$$\frac{dy}{dx} = \left(e^{-\frac{1}{x}}\right)\left(\frac{1}{x^2}\right) > 0 \text{ for } x \neq 0 \Rightarrow y \text{ is increasing}$$

$$\Rightarrow \text{cannot be C or D}$$

$\therefore$ graph must be E.