MA 161-161E EXAM 1 SPRING 1999

Name: Solution Key I.D.#: ____________

Section #: ____________ TA's Name: ____________

1. This package contains 7 pages and 12 problems, problems 1–8 are worth 8 points each and problems 9–12 are worth 9 points each. Correct answers with inconsistent work or no work may not be given credit.

2. Be sure to fill in your name, ID#, Section #, and the name of your recitation instructor.

3. The exam lasts 60 minutes.

4. No books, notes, or calculators, please.

1. The domain of the function $f(x) = \dfrac{\ln|x|}{\sqrt{x+1}}$ is

A. $x > 0$

B. $x > -1$

C. $x > 1$

D. $x > -1,\ x \neq 0$

E. $x \neq 0$

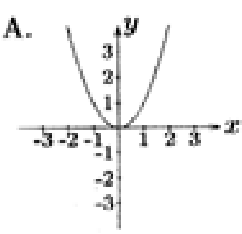
$\dfrac{1}{\sqrt{x+1}} \to x+1 > 0 \to x > -1$

$\ln|x| \to x \neq 0$

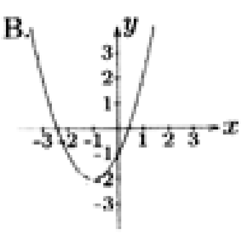
Therefore domain is $x > -1,\ x \neq 0$. Answer is D.

2. The graph of $x^2 - 2x - y = 1$ looks most like

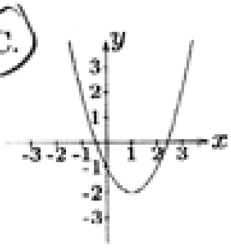
A.



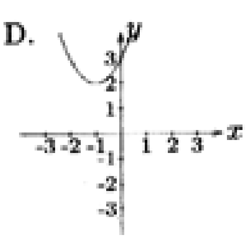
B.



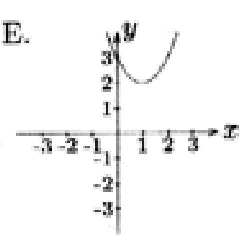
C.



D.



E.



Complete square on x's:

$x^2 - 2x + 1 = y + 1 + 1$

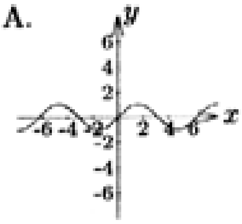
$\to (x-1)^2 = y - (-2)$

$\to$ translate graph of $y = x^2$

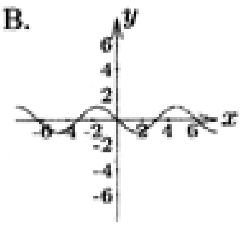
1 unit in x direction, -2 units in y direction. Answer is C.

3. The graph of $y = \cos(\pi - x)$ looks most like

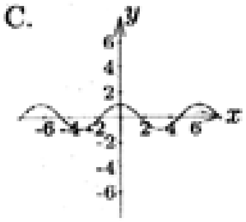
A.



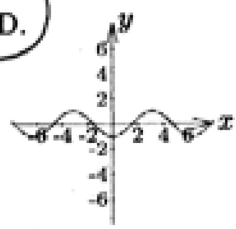
B.



C.



D.



E. None of A, B, C or D.

$y = \cos(\pi - x) = \cos(-(x - \pi)) = \cos(x - \pi)$.

Last equality above because cosine is an even function.
Translate $y = \cos x$ π units to right. Answer is D.

4. $\log_8 2^{-5} =$

A. -40

B. -5

C. $\frac{3}{5}$

D. $-\frac{5}{3}$

E. -15

Let $\log_8 2^{-5} = x$

then $8^x = 2^{-5}$

$\rightarrow (2^3)^x = 2^{-5}$

$\rightarrow 2^{3x} = 2^{-5}$

$\rightarrow 3x = -5$

$\rightarrow x = \frac{-5}{3}$ answer is D.

5. $\lim_{x \to 0} \frac{\tan(2x)\sin x}{x} =$

A. does not exist

B. 1

C. 0

D. 2

E. $\frac{1}{2}$

$= \lim_{x \to 0} \left(\tan(2x)\right)\left(\frac{\sin x}{x}\right)$

$= (0)(1)$

$= 0$ answer is C.

6. The graphs of $f(x) = 3e^{2x}$ and $g(x) = e^x$ meet when $x =$

A. $-\frac{1}{2}\ln 3$

B. $\ln 2$

C. $-\ln 3$

D. $\ln 3$

E. $-\ln(\frac{1}{3})$

$3e^{2x} = e^x$

$\to 3e^{2x} - e^x = 0$

$\to e^x(3e^x - 1) = 0$

$\to 3e^x - 1 = 0$ ($e^x \neq 0$ for all x)

$\to e^x = \frac{1}{3}$

$\to \ln e^x = \ln \frac{1}{3}$

$\to x = \ln \frac{1}{3} = \ln 3^{-1} = -\ln 3$, answer is C.

7. Let $f(x) = x^{2/3}$ then $f'(0)$

A. 0
B. $\frac{2}{3}x^{-1/3}$
C. $\frac{2}{3}$
D. $\frac{1}{3}$
E. does not exist

$$f'(0) = \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0}$$

$$= \lim_{x \to 0} \frac{x^{2/3} - 0}{x - 0}$$

$$= \lim_{x \to 0} \frac{x^{2/3}}{x}$$

$$= \lim_{x \to 0} \frac{1}{x^{1/3}}.$$ This limit does not exist.

Answer is E.

8. Let $f(x) = \sin x + \cos x$ then $f'\left(\frac{\pi}{4}\right) =$

A. 0
B. $2\sqrt{2}$
C. 1
D. $\frac{1}{2} + \frac{\sqrt{3}}{2}$
E. $\sqrt{3}$

$$f'(x) = \cos x - \sin x$$

$$f'\left(\frac{\pi}{4}\right) = \cos\frac{\pi}{4} - \sin\frac{\pi}{4}$$

$$= \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}$$

$$= 0$$

9. Solve the inequality $\dfrac{x+1}{(x-1)(2-x)} > 0$.

$-\infty \quad -1 \quad 1 \quad 2 \quad \infty \quad x$

	$(-\infty,-1)$	$(-1,1)$	$(1,2)$	$(2,\infty)$
$x+1$	$-$	$+$	$+$	$+$
$x-1$	$-$	$-$	$+$	$+$
$2-x$	$+$	$+$	$+$	$-$
$\frac{x+1}{(x-1)(2-x)}$	$+$	$-$	$+$	$-$

$\boxed{x<-1 \text{ or } 1<x<2}$

10. Find an equation of the line that is perpendicular to the line $4x-2y+3=0$ and passes through the point $(3,4)$. Write your answer in the form $ax+by+c=0$ where a, b and c are constants.

$4x-2y+3=0 \rightarrow -2y=-4x-3 \rightarrow y=2x+\frac{3}{2}$.

Given line has slope 2. Therefore perpendicular line has slope $-\frac{1}{2}$.

Perpendicular line has equation: $y-4=-\frac{1}{2}(x-3)$

$\rightarrow 2y-8=-x+3 \rightarrow x+2y-11=0$

$\boxed{x+2y-11=0}$

11. Let $f(x) = \dfrac{2}{x}$. Use the definition of derivative, $f'(a) = \lim\limits_{x \to a} \dfrac{f(x) - f(a)}{x - a}$, to find $f'(2)$.

$$f'(2) = \lim_{x \to 2} \frac{f(x) - f(2)}{x-2} = \lim_{x \to 2} \frac{\frac{2}{x} - 1}{x-2}$$

$$= \lim_{x \to 2} \frac{\frac{2-x}{x}}{x-2} = \lim_{x \to 2} \frac{\overset{-1}{\cancel{2-x}}}{x} \cdot \frac{1}{\underset{1}{\cancel{x-2}}}$$

$$= \lim_{x \to 2} \frac{-1}{x} = -\frac{1}{2}.$$

$f'(2) =$ $-1/2$

12. Find all values of x at which the vertical asymptotes of the graph of $f(x) = \dfrac{(x+2)\ln|x|}{x^2 - 4}$ occur.

$$f(x) = \frac{\ln|x|}{x-2} \quad \text{if } x \neq -2$$

$\lim\limits_{x \to 2} \dfrac{\ln|2|}{x-2} = \pm\infty$ so $x=2$ is a vertical asymptote.

$\lim\limits_{x \to 0} \dfrac{\ln|x|}{x-2} = -\infty$ so $x=0$ is a vertical asymptote,

Vertical asymptotes occur at $x =$ 0, and at $x=2$.