MA 161-161E EXAM 2 SPRING 1999

Name: SOLUTION KEY I.D.#: ____________

Section #: ____________ TA's Name: ____________

1. This package contains 7 pages and 12 problems, problems 1–8 are worth 8 points each and problems 9–12 are worth 9 points each. Correct answers with inconsistent work or no work may not be given credit.

2. Be sure to fill in your name, ID#, Section #, and the name of your recitation instructor.

3. The exam lasts 60 minutes.

4. No books, notes, or calculators, please.

1. Let $f(x) = \dfrac{\sin x}{x^2 - 1}$. Then $f'(2) =$

$$f'(x) = \frac{(\cos x)(x^2-1) - (\sin x)(2x)}{(x^2-1)^2}$$

$$f'(2) = \frac{(\cos 2)(3) - (\sin 2)(4)}{9}$$

$$= \frac{3\cos 2 - 4\sin 2}{9}$$

A. $\dfrac{3\cos 2 - 4\sin 2}{3}$

B. $\dfrac{3\cos 2 + 4\sin 2}{3}$

C. $\dfrac{3\cos 2 - 4\sin 2}{9}$

D. $\dfrac{3\cos 2 + 4\sin 2}{9}$

E. $\dfrac{\cos 2}{4}$

2. Let $f(t) = \ln(\sin(e^t))$. If $t = \ln(\pi/4)$, then $f'(t) =$

$$f'(t) = \frac{1}{\sin e^t}(\cos e^t)(e^t)$$

$$f'(\ln \tfrac{\pi}{4}) = \frac{1}{\sin e^{\ln \frac{\pi}{4}}}\left(\cos e^{\ln \frac{\pi}{4}}\right)\left(e^{\ln \frac{\pi}{4}}\right)$$

$$= \frac{1}{\sin \frac{\pi}{4}}\left(\cos \frac{\pi}{4}\right)\left(\frac{\pi}{4}\right)$$

$$= \frac{1}{\frac{\sqrt{2}}{2}}\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$$

A. $\dfrac{\pi}{4}$

B. $1$

C. $\dfrac{\pi}{2\sqrt{2}}$

D. $0$

E. $\dfrac{\sqrt{2}}{2}$

3. Let $x^2 + 3xy + 2y^2 = 0$; then at the point $(-1, 1)$, $\frac{dy}{dx} =$

Differentiate w.r.t. $x \Rightarrow$

$$2x + 3\left((1)(y) + (x)\left(\frac{dy}{dx}\right)\right) + 4y\frac{dy}{dx} = 0$$

Substitute $(-1,1)$ for $(x,y) \Rightarrow$

$$-2 + 3\left(1 - \frac{dy}{dx}\right) + 4\frac{dy}{dx} = 0$$

$$\Rightarrow -2 + 3 - 3\frac{dy}{dx} + 4\frac{dy}{dx} = 0$$

$$\Rightarrow 1 + \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -1$$

A. $2$

B. $-1$

C. $-\frac{1}{4}$

D. $\frac{1}{4}$

E. $1$

4. A certain population grows exponentially and doubles in 3 days. If the initial population is 100, how long does it take for the population to reach 1200?

Doubles in 3 days : $2A(0) = A(0)e^{3k}$

$\rightarrow 2 = e^{3k} \rightarrow \ln 2 = 3k$

$\rightarrow k = \frac{1}{3}\ln 2.$

$A(t) = 100\, e^{(\frac{1}{3}\ln 2)(t)}$

Solve for $t$ : $1200 = 100\, e^{(\frac{1}{3}\ln 2)(t)}$

$\rightarrow 12 = e^{(\frac{1}{3}\ln 2)(t)}$

$\rightarrow \ln 12 = \left(\frac{1}{3}\ln 2\right)(t)$

$\rightarrow t = 3\frac{\ln 12}{\ln 2}$

A. $\frac{1}{3}\frac{\ln 2}{\ln 12}$

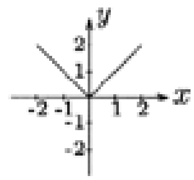
B. $\frac{1}{3}\frac{\ln 12}{\ln 2}$

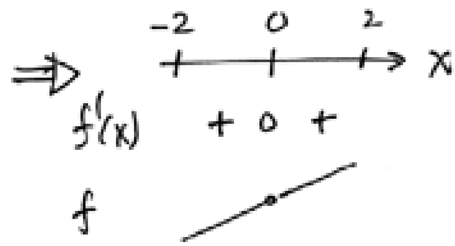
C. $10.5$

D. $3\frac{\ln 12}{\ln 2}$

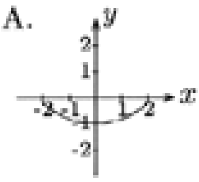
E. $3\frac{\ln 2}{\ln 12}$

5. The following is a graph of $f'(x)$ for $-2 \leq x \leq 2$.




Which of the following could be a graph of $f$?

A.


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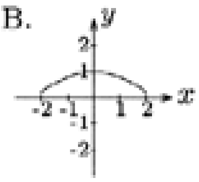
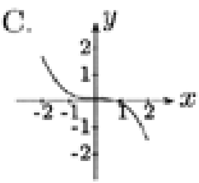
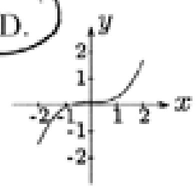


C.


D.


E. There is not enough information to determine a possible graph of $f$.

6. Let $f(x) = x^3 + x^2 - x + 2$. Find all $x$ for which $f$ is decreasing.

$$f'(x) = 3x^2 + 2x - 1$$
$$= (3x - 1)(x + 1)$$
$$= 0 \longrightarrow x = -1, \ 1/3$$

A. $x > -1$

B. $x < \frac{1}{3}$

C. $x < -1$ or $x > \frac{1}{3}$

D. $-1 < x < \frac{1}{3}$

E. $x > \frac{1}{3}$

| | $-\infty$ to $-1$ | $-1$ to $1/3$ | $1/3$ to $\infty$ |
|---|---|---|---|
| $x+1$ | − | + | + |
| $3x-1$ | − | − | + |
| $f'(x)$ | + | − | + |
| | inc. | dec. | inc. |

7. By using a linear approximation, near $x = 27$, the value of $(26)^{2/3}$ is approximately given by

Let $f(x) = x^{2/3}$ ; $f'(x) = \frac{2}{3} x^{-1/3}$

Note: $(26)^{2/3} = (27-1)^{2/3}$

$\approx f(27) + f'(27)\,dx$

$= 27^{2/3} + \frac{2}{3}(27)^{-1/3}\,dx$

$= 9 + \frac{2}{3}\left(\frac{1}{3}\right)(-1)$

$= 9 - \frac{2}{9}$

A. $9 + \frac{1}{3}$

B. $9 - \frac{1}{3}$

C. $9 - \frac{1}{9}$

D. $9 + \frac{2}{9}$

E. $9 - \frac{2}{9}$

8. A spherical balloon is losing air at the rate of 4 cubic inches per minute. What is the rate of change of the radius of balloon when the radius is 10 inches?

Know: $\frac{dV}{dt} = -4$

Want: $\frac{dr}{dt}$ when $r = 10$.

$V = \frac{4}{3}\pi r^3 \longrightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\longrightarrow -4 = 4\pi\, 10^2 \frac{dr}{dt}$

$\longrightarrow \frac{dr}{dt} = \frac{-4}{400\pi} = \frac{-1}{100\pi}$

A. $-\frac{1}{100\pi}$ in/min

B. $\frac{1}{100\pi}$ in/min

C. $-\frac{3}{100\pi}$ in/min

D. $\frac{3}{100\pi}$ in/min

E. $-\frac{\pi}{100}$ in/min

9. Let $f(x) = (e^x + x^3)\cos^2 x$. Find $f'(x)$.

(just write the answer in the box below. use the product rule and chain rule.)

$$\boxed{f'(x) = (e^x + 3x^2)(\cos^2 x) + (e^x + x^3)(2\cos x)(-\sin x)}$$

10. Find all relative extrema of $f(x) = x^3 - 24\ln x$. Justify your answer with the first or second derivative test.

Find critical points: $f'(x) = 3x^2 - 24\left(\frac{1}{x}\right) = \frac{3x^3 - 24}{x}$

Note: $f(x)$ has domain $x > 0$.

$f'(x) = 0 \rightarrow 3x^3 - 24 = 0 \rightarrow x = 2$

| | $-\infty$ | 2 | $\infty$ $x$ |
|---|---|---|---|
| $3x^2-24$ | $-$ | 0 | $+$ |
| $f'(x)$ | $-$ | 0 | $+$ |

f

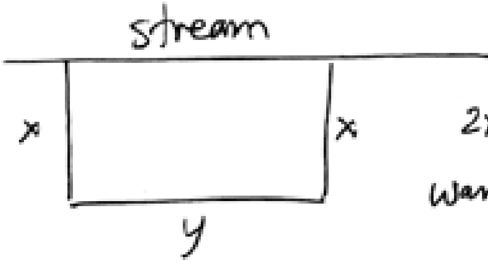
rel min. at $x=2$

No rel max.

rel. max. occur at $x =$ none

rel. min. occur at $x =$ 2

11. A land owner wishes to use 1000 ft of fencing to enclose a rectangular region. Suppose one side of the property lies along a stream and thus needs no fencing. What should the lengths of the sides be in order to maximize the area? Draw a sketch for this problem. Be sure to show that this is a maximum.

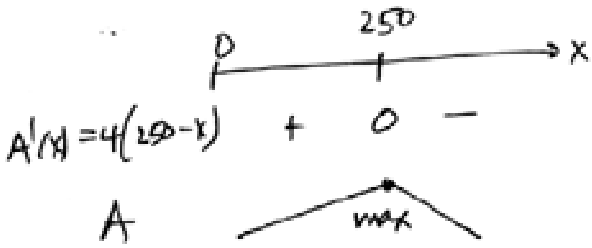



$2x + y = 1000 \longrightarrow y = 1000 - 2x$

want maximum area $= A = xy$

$A(x) = xy = x(1000 - 2x) = 1000x - 2x^2$

$A'(x) = 1000 - 4x = 0 \to x = 250 \to y = 1000 - 2(250) = 500$




length: 500

width: 250

12. Find $f(x)$ if $f'(x) = x^2 + x - 2$ and $f(1) = 1$.

$\Rightarrow f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + C$

$f(1) = 1 = \frac{1}{3} + \frac{1}{2} - 2 + C$

$\to 1 = \frac{2 + 3 - 12}{6} + C$

$\to 1 = -\frac{7}{6} + C \to C = \frac{13}{6}$

$\boxed{f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + \frac{13}{6}}$