

Name: _____

I.D.#: _____

Recitation Instructor: _____ Time of Recitation _____

Lecturer: _____ Section#: _____

Instructions:

1. Fill in your name, student ID number and division and section number on the mark-sense sheet. Also fill out the information requested above.
2. This booklet consists of 9 pages. There are 25 questions, each worth 8 points.
3. Mark your answers on the mark-sense sheet. Please show your working in this booklet.
4. No books, notes or calculators may be used.
5. When you are finished with the exam hand this booklet and the mark-sense sheet, in person, to your instructor.

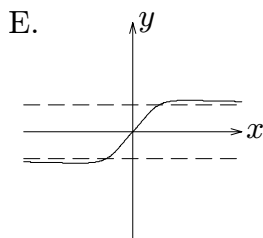
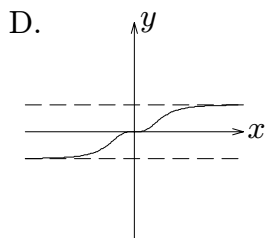
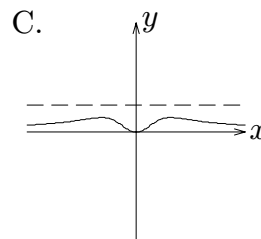
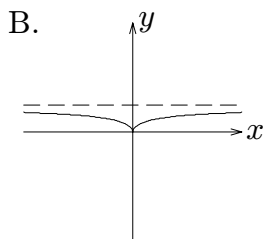
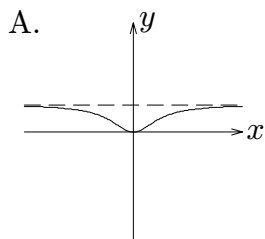
1. The function $f(x) = x^3 - 9x + 1$ has two critical points. The line through these two points has slope

- A. -6
- B. $-\sqrt{3}$
- C. 0
- D. $\sqrt{3}$
- E. 6

2. Evaluate $\lim_{x \rightarrow 0^+} x \csc^2 x =$

- A. $-\infty$
- B. -1
- C. 0
- D. 1
- E. ∞

3. Which could be the graph of $f(x) = \frac{x^3}{1 + |x|^3}$

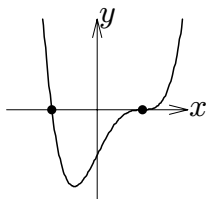


4. What value of a makes the following function continuous at $x = 0$

$$f(x) = \begin{cases} 2 \cos x & x < 0 \\ 3 \sin x + a & x \geq 0 \end{cases}$$

- A. -2
- B. -1
- C. 0
- D. 1
- E. 2

5. $f(x) = (x + 1)^p(x - 1)^q$. The graph of $f(x)$ is



The pair of integers (p, q) could be

- A. $(1, 1)$
- B. $(1, 2)$
- C. $(1, 3)$
- D. $(2, 3)$
- E. $(2, 1)$

6. $F(x) = f(g^3(x) + 1)$ and $f(0) = 8$, $f(9) = 7$, $g(0) = 2$, $f'(0) = 5$, $f'(9) = 2$ and $g'(0) = \frac{5}{6}$. Then $F'(0) =$

- A. 14
- B. 15
- C. 16
- D. 20
- E. 25

7. The functions $x(t)$ and $y(t)$ satisfy the equation $x^3 + y^3 = \frac{9}{2}xy$.
 $x(1) = 2$, $y(1) = 1$ and $x'(1) = -4$. Then $y'(1) =$

- A. -2
- B. -3
- C. -4
- D. -5
- E. -6

8. If $f(x) = \frac{x}{1 + e^x}$ then $f'(1) =$

- A. $\frac{1 + 2e}{(1 + e)^2}$
- B. $\frac{1}{(1 + e)^2}$
- C. $\frac{1 - e}{(1 + e)^2}$
- D. $\frac{1 - 2e}{(1 + e)^2}$
- E. None of the above.

9. Let $F(x) = \int_0^x \sinh^3(t) dt$. Which of the following statements are true:

- I. $F(x)$ is increasing
- II. $F(1) = F(-1)$
- III. $F(1) = -F(-1)$
- IV. F has a minimum at $x = 0$

- A. I and II.
- B. I and III.
- C. II and IV.
- D. III and IV.
- E. II and III.

10. After 5 days $\frac{1}{\sqrt{2}}$ of a sample of a radioactive element remains. The half-life of the element is

- A. 2.5 days
- B. 5 days
- C. $\frac{5}{\ln 2}$ days
- D. 10 days
- E. $\frac{10}{\ln 2}$ days

11. If $f(x) = (\sqrt{x})e^{2x}$ then $\frac{f'(x)}{f(x)} =$

- A. $e^{2x} \left[\ln x + \frac{1}{2x} \right]$
- B. $\frac{1}{\sqrt{x}}e^{2x}$
- C. $\ln x + 1$
- D. $\frac{1}{\sqrt{x}} + e^{2x}$
- E. $2e^{2x}$

12. $\int_0^{\frac{\pi}{2}} \frac{2 \sin x \cos x}{1 + \sin^2 x} dx =$

- A. $\frac{1}{2}$
- B. $\ln 2$
- C. 1
- D. 2
- E. None of the above.

13. The area between the graph of $y = x^2$ and $y = \sqrt{8x}$ is

A. $\int_0^2 (\sqrt{8x} - x^2) dx$

B. $\int_0^2 (x^2 - \sqrt{8x}) dx$

C. $\int_0^1 (\sqrt{8x} - x^2) dx$

D. $\int_0^1 (x^2 - \sqrt{8x}) dx$

E. None of the above

14. If $f(x) = x^2 \tan^{-1} x$ then $f'(1) =$

A. $\frac{\pi}{4}$

B. 1

C. $\frac{\pi}{2} + \frac{1}{2}$

D. $1 + \frac{\pi}{4}$

E. $\left(\frac{\pi}{4}\right)^2$

15. $\int_0^1 7^x dx =$

A. 6

B. $6 \ln 7$

C. 7

D. $\frac{6}{\ln 7}$

E. $7 \ln 7$

16. $\int_0^4 \frac{1}{16+x^2} dx =$

A. $\frac{\pi}{16}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. π

E. None of the above.

17. $\int \frac{\cosh x}{\sqrt{1-\sinh^2 x}} dx$

A. $\sin^{-1}(\sinh(x)) + C$

B. $2\sqrt{1-\sinh^2 x} + C$

C. $\ln|\cosh x| + C$

D. $\sinh(\sqrt{1+x^2}) + C$

E. None of the above

18. If $F(x) = \int_{\sin x}^0 e^{t^2} dt$, then $F'(x) =$

A. $-2e^{\sin^2 x} \sin x$

B. $-e^{x^2} \cos x$

C. $-e^{\sin^2 x}$

D. $-e^{\cos^2 x}$

E. $-e^{\sin^2 x} \cos x$

19. $\lim_{x \rightarrow 0^-} \frac{\cos x}{\ln|1+x|} =$

A. $-\infty$

B. -1

C. 0

D. 1

E. ∞

20. Let $f''(x) = (x - 1)x^2(x + 1)^3$. Then the inflection points of f occur when

- A. $x = 0$
- B. $x = -1$
- C. $x = -1, 1$
- D. $x = -1, 0, 1$
- E. f has no inflection points

21. Let $g'(x) = (x - 1)x^4(x + 1)^5$. The critical numbers of g are $x = -1, 0, 1$. g has

- A. one relative maximum and two relative minima
- B. one relative minimum and two relative maxima
- C. one relative minimum and one relative maximum
- D. one relative maximum and no relative minimum
- E. one relative minimum and no relative maximum

22. $\lim_{x \rightarrow -1^+} \frac{|x| - 1}{x + 1} =$

- A. -2
- B. -1
- C. 0
- D. 1
- E. does not exist

23. $\int_{-1}^1 x^3 \sin(x^4) dx =$

- A. $2 \cos 1$
- B. $1 + \cos(1)$
- C. 0
- D. $\cos(1) + \cos(-1)$
- E. 2

24. Which of the following is a horizontal asymptote of
 $f(x) = \sqrt{x^2 + 4x + 3} - x$

- A. 0
- B. 1
- C. 2
- D. 3
- E. 4

25. Sand is falling into a conical pile at a rate of 2 cubic feet per second. The height of the cone is always two-thirds of the radius of its base. Find the rate of change of the radius of the pile when it contains 6π cubic feet of sand. ($V = \frac{1}{3}\pi r^2 h$)

- A. $\frac{\pi}{3}$
- B. $\frac{\pi}{2}$
- C. $\frac{1}{3\pi}$
- D. $\frac{2}{3\pi}$
- E. $\frac{1}{2}$