MA 161–161E **FINAL EXAM** **May 7, 1999**

Name: ______________________________

ID #: ______________________________

Recitation Instructor ________________ Time of Recitation ________________

Section #: ____________________________

Instructions:

1. Fill in your name, student ID number and division and section numbers on the mark–sense sheet. Also fill in the information requested above.
2. This booklet consists of 10 pages. There are 25 questions, each worth 8 points. Each question has exactly one correct answer.
3. Mark your answers on the mark–sense sheet. Please show your work in this booklet.
4. No books, notes or calculators please.
5. When you are finished with the exam, hand this booklet and the mark–sense sheet, in person, to your instructor.

1. $\tan(\sin^{-1}(x)) =$

A. $\dfrac{1}{\sqrt{1-x^2}}$

B. $\dfrac{1}{\sqrt{1+x^2}}$

C. $\dfrac{x}{\sqrt{1+x^2}}$

D. $\dfrac{1}{1+x^2}$

E. $\dfrac{x}{\sqrt{1-x^2}}$

2. Let $f(x) = \dfrac{1}{x-2}$ and $g(x) = \dfrac{1}{x-1}$. Find the domain of $f \circ g$.

A. $x \neq 1$ and $x \neq 2$

B. $x \neq 1$ and $x \neq \frac{3}{2}$

C. $x \neq 2$ and $x \neq \frac{3}{2}$

D. $x \neq 1$ and $x \neq 2$ and $x \neq \frac{3}{2}$

E. $x \neq \frac{3}{2}$

3. The graph of $y = x^2 + 4x + 2$ can be drawn by translating the graph of $y = x^2$ so that the vertex (lowest point) of the graph of $y = x^2$ is translated to the point

A. $(2, 2)$

B. $(-2, 2)$

C. $(2, -2)$

D. $(-2, -2)$

E. $(-2, 0)$

4. Suppose $\lim_{x \to 1} f(x) = 3$. Which of the following could be the graph of f?

A. only 1

B. only 1 and 2

C. only 1 and 2 and 3

D. 1 and 2 and 3 and 4

E. only 3

5. Let $f(x) = \dfrac{x^2 - x - 6}{x + 2}$. What value of f at $x = -2$ makes f continuous at $x = -2$?

A. 3

B. 2

C. 0

D. -2

E. $\lim_{x \to -2} f(x)$

6. If $f(x) = \dfrac{x^2 - 3x}{2x + 1}$, then $f'(1) =$

A. $\frac{1}{9}$

B. $-\frac{2}{3}$

C. $\frac{7}{9}$

D. $-\frac{7}{9}$

E. $\frac{1}{3}$

7. The x-intercept of the line tangent to the graph of $y = 3x^2 - 4x$ at the point $(1, -1)$ is

   A. $x = -2$

   B. $x = \frac{3}{2}$

   C. $x = 0$

   D. $x = \frac{1}{2}$

   E. $x = 2$

8. If $f(x) = \sqrt{\sin(x^3 + 2x)}$, then $f'(1) =$

   A. $\dfrac{5\cos 3}{2\sqrt{\sin 3}}$

   B. $\dfrac{\cos 3}{2\sqrt{\sin 3}}$

   C. $\dfrac{5\sin 3}{2\sqrt{\cos 3}}$

   D. $\dfrac{\sin 3}{2\sqrt{\cos 3}}$

   E. $5\tan 3$

9. Let $y^2 + y = 4x^2 + 2\sin\left(\dfrac{\pi x}{6}\right) + 1$ define y implicitly as a function of x near $(1, 2)$. Then at $x = 1$, $\dfrac{dy}{dx} =$

   A. $\dfrac{1}{5}\left(\dfrac{\pi}{6}\right)$

   B. $\dfrac{1}{3}\left(8 + \dfrac{\sqrt{3}}{2}\right)$

   C. $8 + \pi\dfrac{\sqrt{3}}{2}$

   D. $\dfrac{1}{5}\left(8 + \dfrac{\pi\sqrt{3}}{6}\right)$

   E. $\dfrac{1}{3}\left(8 + \dfrac{\pi}{6}\right)$

10. A substance decays so that $P'(t) = kP(t)$. One third of the substance is lost in 2 hours. How many hours (from the starting time) will it take until only half the substance remains?

A. $2\,\dfrac{\ln\left(\frac{1}{2}\right)}{\ln\left(\frac{3}{2}\right)}$ hours

B. $2\,\dfrac{\ln 2}{\ln\left(\frac{3}{2}\right)}$ hours

C. $2\,\dfrac{\ln 2}{\ln 3}$ hours

D. $\dfrac{\ln 2}{\ln\left(\frac{3}{2}\right)}$ hours

E. $\dfrac{\ln(2)}{\ln(3)}$ hours

11. The function $f(x) = \dfrac{1}{2}x^2 - \ln x$

A. is increasing on $(0, \infty)$

B. has a local minimum at $x = 0$

C. has a local minimum at $x = 1$

D. has a local minimum at $x = \pm 1$

E. has an inflection point at $x = 1$

12. Shown below is the graph of $f'(x)$. Then the graph of f is concave down on the interval(s).

A. $[-2.0]$ and $[0, 2]$

B. $[-2, -1]$ and $[0, 1]$

C. $[-2, -1]$

D. $[-1, 0]$ and $[1, 2]$

E. $[-2, 0)$ and $(0, 2]$

13. Let a box with no top have height h and a square base with each side of length x. If the box has volume V then the surface area of the box is

A. x^2h

B. $\pi x^2 h$

C. $x^2 + \frac{4V}{x}$

D. $2x^2 + 4V$

E. $x^2 + 2xh$

14. The function $f(x) = \sin x - \dfrac{\sqrt{3}}{2} x$

A. has a local minimum at $x = \frac{\pi}{6}$

B. is always decreasing for $x > 0$

C. has a local minimum at $x = \frac{\pi}{3}$

D. has a local maximum at $x = \frac{\pi}{3}$

E. has a local maximum at $x = \frac{\pi}{6}$

15. The graph shown below is most similar to that of the function

A. $y = \dfrac{1-x}{x^2(x-2)}$

B. $y = \dfrac{x-1}{x^2(x-2)}$

C. $y = \dfrac{x-1}{x(x-2)}$

D. $y = \dfrac{x^2}{(x-1)(x-2)}$

E. $y = \dfrac{(1-x)^2}{x^2(x-2)}$

16. Let a rectangle be constructed in the first quadrant with one vertex at $(0,0)$ and the opposite vertex at the point (x, e^{-x}). Let $A(x)$ be the area of this rectangle. Then the first derivative $A'(x)$ is

A. $-e^{-x}$

B. $e^{-x}(1-x)$

C. $e^{x}(x-1)$

D. $1-e^{-x}$

E. xe^{-x}

17. $\displaystyle\int_{-1}^{1} \frac{d}{dx}\sqrt{1+x^3}\,dx =$

A. $\sqrt{2}$

B. $\dfrac{3}{2\sqrt{2}}$

C. 0

D. $\dfrac{3}{\sqrt{2}}$

E. $2\sqrt{2}$

18. $\displaystyle\int_{0}^{1} 9(x^2+3)^8 x\,dx =$

A. $\frac{1}{2}\,4^9$

B. $4^9 - 3^9$

C. $\frac{1}{2}\,(4^9 - 3^9)$

D. 3^9

E. $4^8 - 3^8$

19. The area enclosed by the graphs of $y = x^2$ and $x + y = 2$ is equal to

A. $\displaystyle\int_{-1}^{2} (2 - x - x^2)dx$

B. $\displaystyle\int_{-1}^{2} (x^2 + x - 2)dx$

C. $\displaystyle\int_{-2}^{1} (x^2 + x - 2)dx$

D. $\displaystyle\int_{-2}^{1} (2 - x - x^2)dx$

E. $\displaystyle\int_{-2}^{1} (x^2 + x + 2)dx$

20. If $f(x) = x^3 + 3x - 1$ then the derivative of its inverse at 3, $(f^{-1})'(3)$ is equal to

A. 6

B. -6

C. $\frac{1}{6}$

D. $-\frac{1}{6}$

E. $\frac{1}{3}$

21. $\dfrac{d}{dx}\,(x)^{\sin x} =$

A. $x^{\sin x} \cos x \ln x$

B. $x^{\sin x - 1}$

C. $x^{\sin x} \dfrac{\sin x}{x}$

D. $x^{\sin x} \left\{ \cos x \ln x + \dfrac{\sin x}{x} \right\}$

E. $x^{\cos x} + x^{\sin x - 1}$

22. $\displaystyle\int_0^1 5^x dx =$

A. $\dfrac{4}{\ln 5}$

B. $\dfrac{5}{\ln 5}$

C. 5

D. 1

E. $4 \ln 5$

23. $\displaystyle\int_0^{\frac{1}{2}} \frac{1}{4x^2+1}\,dx =$

A. 1

B. π

C. $\frac{\pi}{2}$

D. $\frac{\pi}{4}$

E. $\frac{\pi}{8}$

24. $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin^3 x}\,dx =$

A. 1

B. $-\frac{1}{2}$

C. 0

D. $\frac{1}{2}$

E. 1

25. Let $f(x) = \sin^{-1}(x^2)$ then $f'(x) =$

A. $\dfrac{1}{\sqrt{1-x^2}}$

B. $\dfrac{1}{\sqrt{1-x^4}}$

C. $\dfrac{2x}{1+x^4}$

D. $\dfrac{1}{1+x^4}$

E. $\dfrac{2x}{\sqrt{1-x^4}}$