MA 161 Midterm September 1997

Name: Solution Key I.D.#: ______

Section #: ______ TA's Name: ______

1. This package contains 8 pages and 14 problems, each worth 7 points. Mark your answers on the answer sheet, using a #2 pencil. Turn in both this package and your answer sheet to your recitation instructor.

2. Be sure to fill in your name, ID#, Section #, and the name of your recitation instructor.

3. The exam lasts 60 minutes.

4. No books, notes, or calculators, please.

1. The inequality $(x-1)(x+2)(x-3) < 0$ is solved by

$-\infty \quad -2 \quad 1 \quad 3 \quad \infty \quad x$

$x-1$	$-$	$-$	$+$	$+$
$x+2$	$-$	$+$	$+$	$+$
$x-3$	$-$	$-$	$-$	$+$
$(x+2)(x-3)$	$-$	$+$	$-$	$+$

A. $x \le -2$ or $1 < x < 3$

B. $x < -2$ or $1 < x < 3$

C. $-2 < x < 1$ or $x > 3$

D. $x < -3$ or $-1 < x < 2$

E. $-3 < x < -1$ or $x > 2$

$(x-1)(x+2)(x-3) < 0 \rightarrow x < -2$ or $1 < x < 3$

2. The solutions of the equation $|x| = |1 + 2x|$ are

either $x = 1 + 2x$ or $-x = 1 + 2x$

$x = 1 + 2x \rightarrow -x = 1 \rightarrow x = -1$

$-x = 1 + 2x \rightarrow -3x = 1 \rightarrow x = -\frac{1}{3}$

A. $x = 1$ and $x = \frac{1}{3}$

B. $x = -1$ and $x = -\frac{1}{3}$

C. $x = -1$ and $x = \frac{2}{3}$

D. $x = -2$ and $x = \frac{1}{3}$

E. None of the above

Let ℓ:

3. An equation of the line through $(1, 2)$ and perpendicular to the line $3x + 2y = 4$ is

$\ell: \rightarrow 2y = -3x + 4 \rightarrow y = -\frac{3}{2}x + 2$

$\rightarrow$ slope is $-3/2$

Perpendicular slope is $\frac{2}{3}$

A. $-3x + 2y - 4 = 0$

B. $3x + 6y - 8 = 0$

C. $-2x + 3y = 4$

D. $x - 3y = -5$

E. None of the above

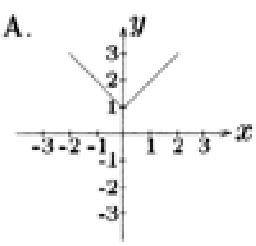
Perpendicular line is: $y - 2 = \frac{2}{3}(x - 1)$

$\rightarrow 3y - 6 = 2x - 2$

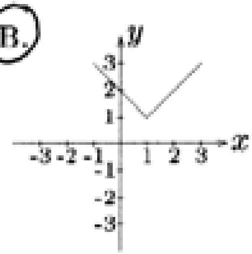
$\rightarrow -2x + 3y = 4$

4. The graph of the function $f(x) = 1 + |x - 1|$ looks most like

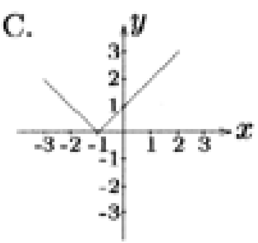
A.



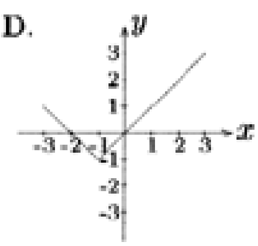
B.



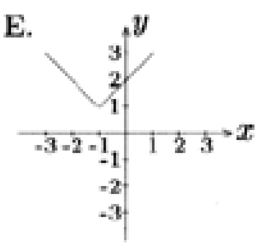
C.



D.



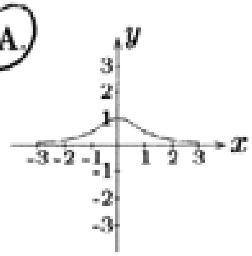
E.

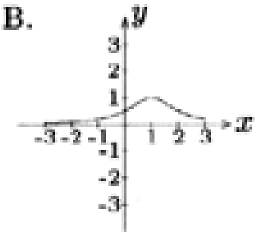


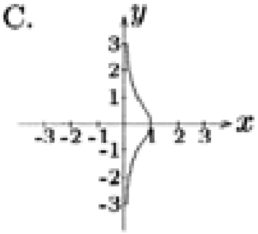
$\rightarrow f(x) - 1 = |x - 1|$

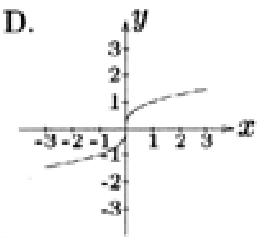
$\rightarrow$ translate graph of $y = |x|$ 1 unit up and 1 unit to right.

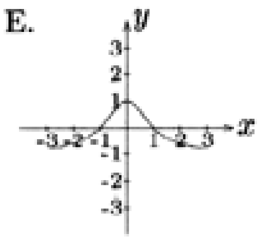
5. The graph of the function $f(x) = \dfrac{1}{1+x^2}$ looks most like:

A.


B.


C.


D.


E.


f is an even function, i.e., $f(-x) = f(x)$.

$\rightarrow$ possible choices are A and E.

$f(x) = \dfrac{1}{1+x^2} > 0$ for all x

$\rightarrow$ A must be graph.

6. If $f(x) = \sqrt{9-x^2}$ and $g(x) = \sqrt{x-1}$, the domain of the product $f(x)g(x)$ is

A. $1 \le x \le 3$

B. $1 < x < 3$

C. $-3 \le x \le -1$

D. $-3 < x < 1$

E. $-3 \le x \le 1$

domain of f is $9 - x^2 \geq 0$

$\rightarrow x^2 \le 9$

$\rightarrow -3 \le x \le 3$

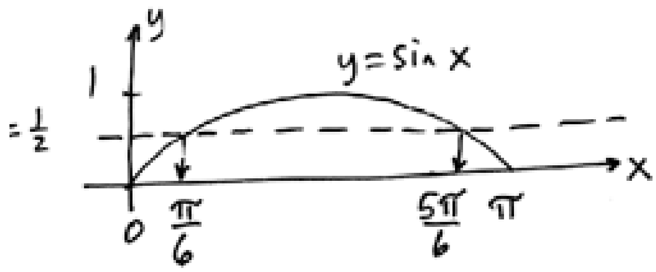
domain of g is $x - 1 \geq 0$

$\rightarrow x \geq 1$

domain of $f(x)g(x)$ is intersection of domain of f and g

$\rightarrow 1 \le x \le 3$

7. Solve the inequality $\sin x > \dfrac{1}{2}$ for x in $[0, \pi]$.



graph of $y = \sin x$ is above
graph of $y = \frac{1}{2}$ for $\frac{\pi}{6} < x < \frac{5\pi}{6}$

A. $\dfrac{\pi}{4} < x < \dfrac{3\pi}{4}$

B. $\dfrac{\pi}{6} < x < \dfrac{\pi}{3}$

C. $\dfrac{\pi}{2} < x$

D. $\dfrac{\pi}{6} < x < \dfrac{5\pi}{6}$

E. $\dfrac{\pi}{3} < x < \dfrac{2\pi}{3}$

8. Simplify $\dfrac{3^{\sqrt{2}}9^{\sqrt{2}}}{3^{3\sqrt{2}-1}}$.

$$= (3^{\sqrt{2}})(3^2)^{\sqrt{2}}(3^{-3\sqrt{2}+1})$$

$$= 3^{\sqrt{2}+2\sqrt{2}-3\sqrt{2}+1}$$

$$= 3^1$$

A. 3

B. $\dfrac{1}{3}$

C. $3^{\sqrt{2}}$

D. 2

E. None of the above

9. If the position of a particle at time t is given by $f(t) = 16t^2$, its velocity at time $t = 1$ is

$$\text{velocity} = \frac{df}{dt} = 16(2t) = 32t$$

$$v(1) = 32$$

A. 16
B. −16
C. 32
D. −32
E. 64

10. $\lim_{x \to -3} \frac{x^2 - 9}{x^2 + 3x} =$ $\frac{0}{0}$

$$= \lim_{x \to -3} \frac{(x-3)\cancel{(x+3)}}{(x)\cancel{(x+3)}} = \frac{-6}{-3} = 2.$$

A. 0
B. 1
C. 2
D. 3
E. −3

11. Let $f(x) = \dfrac{3x-3}{x-1}$ and $g(x) = \dfrac{x^3-1}{x^2+2x+1}$. Which of the following statements is true?

A. Neither f nor g is continuous at $x = 1$

B. Only f is continuous at $x = 1$

C. Only g is continuous at $x = 1$

D. Both f and g are continuous at $x = 1$

E. Not enough information

Continuity of h at $x=1$ means $\lim_{x\to 1} h(x) = h(1)$.

$f(1)$ does not exist, hence f not continuous at $x=1$.

$g(1) = 0 = \lim_{x\to 1} g(x)$, hence g is continuous at $x=1$.

12. $\lim_{x\to 0} x^2 \cos \dfrac{2}{x} =$

A. 0

B. 1

C. 2

D. $\dfrac{1}{4}$

E. limit does not exist

Note: $\lim_{x\to 0} x^2 = 0$

and $\lim_{x\to 0} \cos\frac{2}{x}$ does not exist.

Use the squeeze theorem:

$-1 \le \cos\frac{2}{x} \le 1$ for all $x \ne 0$

$-x^2 \le x^2 \cos\frac{2}{x} \le x^2$ since $x^2 > 0$ if $x \ne 0$

$\lim_{x\to 0} -x^2 = \lim_{x\to 0} x^2 = 0$, so by squeeze thm, $\lim_{x\to 0} x^2\cos\frac{2}{x} = 0$

13. $\lim_{x\to 2^-}\left(\frac{1}{x-2}-\frac{2}{x^2-4}\right) =$ $-\infty + \infty$

A. 0

B. 1

C. ∞

D. $-\infty$

E. -1

Note: $\frac{1}{x-2} - \frac{2}{x^2-4}$

$= \frac{1}{x-2}\cdot\frac{x+2}{x+2} - \frac{2}{x^2-4}$

$= \frac{x+2-2}{x^2-4} = \frac{x}{x^2-4}$

and $\lim_{x\to 2^-} \frac{x}{x^2-4} = \frac{2}{0^-} = -\infty$

14. Let $f(x) = \frac{2}{x}$ then $f'(1)$ is

A. -2

B. -1

C. 0

D. 2

E. Does not exist

$f(x) = 2x^{-1}$

$\rightarrow f'(x) = 2(-x^{-2}) = -2x^{-2}$

$\rightarrow f'(1) = -2(1)^{-2} = -2$