MA 161 & 161E **EXAM 1** **FEBRUARY 9, 2000**

Name: Solution Key

I.D. #:

Rec. Instructor: **Time of Rec. Sect.:**

Lecturer:

Instructions:

1. On the mark sense sheet
   a. Fill in instructor's name and course number.
   b. Fill in your name, student identification number and division and section number, and fill in the appropriate spaces with a pencil.
   c. Fill in the appropriate letter on your mark–sense answer sheet.
   d. Hand in both the answer and question booklet to your recitation instructor when you are done.
2. Verify that you have all the pages (there are 8 pages).
3. Calculators are not allowed.
4. Circle the letter of your response to each question.

1. Solve the inequality $\frac{(x-1)(x+3)}{x-2} > 0$ for x.

A. $-3 < x < 2$
B. $x < -3$ or $1 < x < 2$
C. $x < -3$ or $x > 1$
D. $-3 < x < 1$
E. $-3 < x < 1$ or $x > 2$

	$-\infty$ to -3	-3 to 1	1 to 2	2 to ∞
$x-1$	$-$	$-$	$+$	$+$
$x+3$	$-$	$+$	$+$	$+$
$x-2$	$-$	$-$	$-$	$+$
$\frac{(x-1)(x+3)}{x-2}$	$-$	$+$	$-$	$+$

$\frac{(x-1)(x+3)}{x-2} > 0 \longleftrightarrow -3 < x < 1$ or $x > 2$

2. Find an equation of the line that is perpendicular to $3x+2y+4=0$ and that contains the point $(-2,1)$.

A. $3x-2y+8=0$
B. $2x-3y+7=0$
C. $2x+3y-1=0$
D. $-2x+3y+9=0$
E. $3x+2y+4=0$

$3x+2y+4=0$ has slope $-\frac{3}{2}$

perpendicular line has slope $\frac{2}{3}$

and equation: $y-1=\frac{2}{3}(x+2)$

$\longrightarrow 3y-3=2x+4$

$\longrightarrow -2x+3y-7=0$

$\longrightarrow 2x-3y+7=0$

3. Find the domain of the function $y = \dfrac{3 - \ln(x+2)}{\sqrt{1-x}}$.

A. $x > -2$

B. $x > 1$

C. $x < 1$

D. $-2 < x < 1$

E. all real numbers x

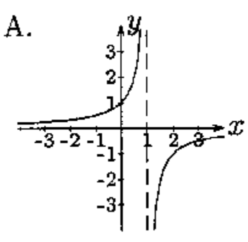
denominator: $\sqrt{1-x} \rightarrow 1-x > 0 \rightarrow x < 1$

neumerator: $\ln(x+2) \rightarrow x+2 > 0 \rightarrow x > -2$

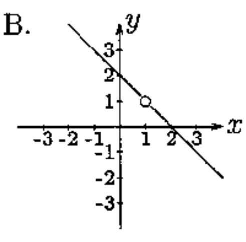
domain is intersection of $x < 1$ and $x > -2$ $\rightarrow -2 < x < 1$

4. Which of the following is the graph of $y = \dfrac{x^2+x-2}{x-1}$?

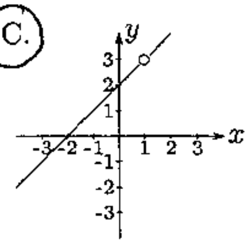
A.



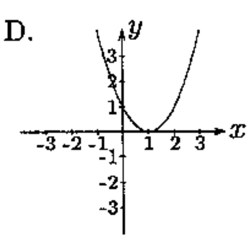
B.



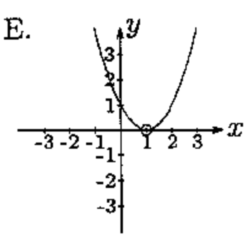
C.



D.



E.



$$\frac{x^2+x-2}{x-1} = \frac{(x-1)(x+2)}{x-1}$$

$$= x+2 \text{ if } x \neq 1$$

5. Find the domain for $f \circ g$ where $f(x) = \dfrac{1}{x+2}$ and $g(x) = \dfrac{1}{x-1}$.

A. $x \neq -2$

B. $x \neq 1$

C. $x \neq -2$ and $x \neq 1$

D. $x \neq \frac{1}{2}$ and $x \neq 1$

E. $x \neq -2$ and $x \neq \frac{1}{2}$

domain of g is $x \neq 1$.

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x-1}\right)$$

$$= \frac{1}{\frac{1}{x-1} + 2}$$

$$= \frac{1}{\frac{1 + 2(x-1)}{x-1}}$$

$$= \frac{x-1}{2x-1}$$ → domain of $f \circ g$ is $x \neq \frac{1}{2}$ and $x \neq 1$.

6. Solve $\sqrt{3} \sin x < \cos x$ for $0 \leq x < \pi$.

A. $\frac{\pi}{6} \leq x < \frac{\pi}{2}$

B. $\frac{\pi}{3} \leq x < \frac{\pi}{2}$

C. $0 \leq x < \frac{\pi}{6}$

D. $0 \leq x < \frac{\pi}{3}$ or $\frac{\pi}{2} < x < \frac{5\pi}{6}$

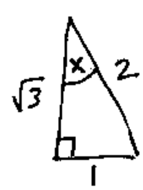
E. none of the above

intersection of $y = \sqrt{3} \sin x$ and $y = \cos x$:

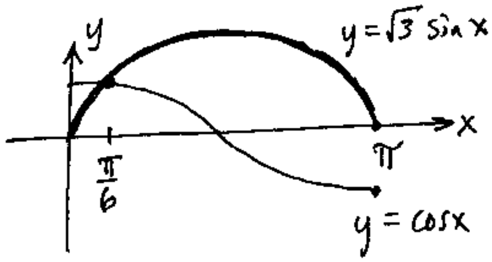
$\sqrt{3} \sin x = \cos x$

$\rightarrow \tan x = \frac{1}{\sqrt{3}}$

$\rightarrow x = \frac{\pi}{6}$



Consider graphs of $\sqrt{3} \sin x$ and $\cos x$:



→ graph of $y = \sqrt{3} \sin x$ below graph of $y = \cos x$ for $0 \leq x < \frac{\pi}{6}$

7. $\log_{\frac{1}{9}} 3^x$ is equal to

A. $-\frac{x}{2}$
B. 3
C. x
D. e
E. $\frac{x}{2}$

Let $y = \log_{\frac{1}{9}} 3^x$,

then $\left(\frac{1}{9}\right)^y = 3^x$

$\rightarrow (9^{-1})^y = 3^x$

$\rightarrow (3^{-2})^y = 3^x$

$\rightarrow 3^{-2y} = 3^x$

$\rightarrow -2y = x \rightarrow y = -\frac{x}{2}$

8. The slope of the line tangent to the graph of $f(x) = 2x^2 - 3x$ at $(2, 2)$ is

A. 4
B. 5
C. $\frac{0}{0}$
D. 3
E. cannot be determined

Slope of tangent line is derivative.

$f'(x) = 4x - 3$

$f'(2) = 8 - 3 = 5$

9. $\lim_{x\to 2}\left[(x-2)^2+3\right]\sin\left(\frac{\pi}{x^2-2}\right)$ is

A. 0
B. 1
C. -1
D. 3
E. 2

$= \left[(2-2)^2+3\right]\sin\left(\frac{\pi}{2^2-2}\right)$ by continuity

$= [0+3]\sin\left(\frac{\pi}{2}\right)$

$= (3)(1)$

$= 3$

10. $\lim_{r\to -2^-}\frac{|r+2|}{r+2}$ is

A. ∞
B. $-\infty$
C. 0
D. 1
E. -1

Note: $r \to -2^- \Rightarrow r < -2 \Rightarrow r+2 < 0$

$\Rightarrow \frac{|r+2|}{r+2} = \frac{-(r+2)}{r+2} = -1$ provided $r \neq -2$

Therefore $\lim_{r\to -2^-}\frac{|r+2|}{|r+2|} = \lim_{r\to -2^-}(-1) = -1$

11. Let $f(x) = x^2 - 3x + 4$. Find the point on the graph of $y = f(x)$ where the line tangent to the graph is parallel to the line $x + y = 1$.

A. $(-2, 14)$
B. $(-1, 8)$
C. $(0, 4)$
D. $(1, 2)$
E. $(2, 2)$

$x + y = 1$ has slope -1

$f'(x) = 2x - 3$ is slope of tangent line.

We want $2x - 3 = -1$

$\rightarrow 2x = 2$

$\rightarrow x = 1$

$\rightarrow f(1) = (1)^2 - 3(1) + 4 = 2$

Desired point is $(1, 2)$

12. Let $f(x) = \dfrac{e^x}{x^3}$. Then $f'(1)$ is

A. 0
B. $-e$
C. $-2e$
D. $-3e$
E. $-4e$

Use the quotient rule. Find $f'(x)$ and then evaluate $f'(1)$.

$$f'(x) = \frac{(e^x)(x^3) - (e^x)(3x^2)}{(x^3)^2}$$

$$f'(1) = \frac{e - 3e}{1} = -2e.$$

13. Let $f(x) = -3x^2 + \frac{1}{3x^3}$. Then $f'(x)$ is

Note: $f(x) = -3x^2 + 3x^{-3}$

Then, $f'(x) = -3(2x) + \frac{1}{3}(-3x^{-4})$

$= -6x - x^{-4}$

$= -6x - \frac{1}{x^4}$

A. $-3x + \frac{1}{x^4}$

B. $-6x^3 + \frac{1}{9x^2}$

C. $-6x - \frac{1}{x^4}$

D. $-6x + \frac{1}{3x^2}$

E. $-3x - \frac{1}{x^2}$

14. The $\lim_{x \to 3} \frac{xe^x - 3e^3}{x - 3}$ is the derivative of a function at $x = 3$. Its value is

$\lim_{x \to 3} \frac{xe^x - 3e^3}{x-3} = \lim_{x \to 3} \frac{f(x) - f(3)}{x-3}$

where $f(x) = xe^x$.

$f'(x) = (1)(e^x) + (x)(e^x)$

$f'(3) = e^3 + 3e^3 = 4e^3$

A. $4e^3$

B. $2e^3$

C. 0

D. $-2e^3$

E. $-4e^3$

15. Let a be a constant and

$$f(x) = \begin{cases} 2x+1, & x \geq 1 \\ a(x^2+2x-3)+3, & x < 1. \end{cases}$$

Choose a so that $f(x)$ is differentiable at $x = 1$. Then a is

A. $a = 1$

B. $a = \frac{1}{2}$

C. $a = \frac{1}{4}$

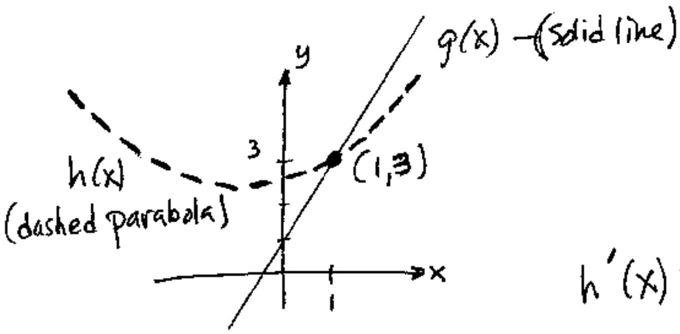
D. $a = \frac{1}{8}$

E. $a = 0$

Here's the shorter solution (longer one on next page):

Note: $a(x^2+2x-3)+3 = 3$ for all a and $x=1$.

Hence graphs of $g(x) = 2x+1$ and $h(x) = a(x^2+2x-3)+3$ always intersect at $(1,3)$. Note also in figure to left that derivatives of g and h do not agree at $x=1$ (wrong "a"!)



$h'(x) = a(2x+2) \rightarrow h'(1) = 4a$

$g'(x) = 2$

Since we want the derivatives of g and h to agree at $x=1$, $4a = 2 \rightarrow a = \frac{1}{2}$

Note: The graph of f is made up of the part of the graph of h, $x<1$ and the part of the graph of g, $x \geq 1$.

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EXAM 1
SPRING, 2000

Problem 15 (slightly longer solution):

We want $f'(1)$ to exist.

Thus $f'(1) = \lim_{x \to 1} \frac{f(x) - f(1)}{x-1}$ must exist.

Thus the two one-sided limits,

$$\lim_{x \to 1^-} \frac{f(x) - f(1)}{x-1} \quad \text{and} \quad \lim_{x \to 1^+} \frac{f(x) - f(1)}{x-1}$$

must both exist <u>and</u> have the same value.

Note that $f(1) = 2(1) + 1 = 3$.

<u>CASE 1</u> For $x > 1$, $f(x) = 2x + 1$, and therefore

$$\lim_{x \to 1^+} \frac{f(x) - f(1)}{x-1} = \lim_{x \to 1^+} \frac{2x+1-3}{x-1} = \lim_{x \to 1^+} \frac{2x-2}{x-1}$$

$$= \lim_{x \to 1^+} \frac{2(x-1)}{x-1} = \lim_{x \to 1^+} 2 = 2.$$

<u>CASE 2</u> For $x < 1$, $f(x) = a(x^2 + 2x - 3) + 3$, and

$$\lim_{x \to 1^-} \frac{f(x) - f(1)}{x-1} = \lim_{x \to 1^-} \frac{a(x^2+2x-3) + 3 - 3}{x-1}$$

$$= \lim_{x \to 1^-} \frac{a(x-1)(x+3)}{x-1}$$

$$= \lim_{x \to 1^-} a(x+3) = 4a.$$

Finally, $\lim_{x \to 1} \frac{f(x) - f(1)}{x-1}$ exists when $4a = 2 \rightarrow a = \frac{1}{2}$.