1. The function $f(x) = x^4 - 8x^3 + 24x^2 - 7\pi$ has

   A. no inflection points

   B. two inflection points at $x = 1$ and $x = 4$

   C. an inflection point at $x = 2$

   D. an inflection point at $x = 0$

   E. two inflection points at $x = 0$ and $x = 4$

$$f'(x) = 4x^3 - 24x^2 + 48x$$
$$f''(x) = 12x^2 - 48x + 48$$
$$= 12(x-2)^2$$

$f''(x) > 0$ for $x \neq 2$ $\Rightarrow$ f concave up $(x \neq 2)$

$\Rightarrow$ f has no inf. pts.

2. The limit $\lim\limits_{x\to\infty} \dfrac{2 - 3x - 4x^2}{10 + 6x + 3x^2}$.

   A. does not exist

   B. $= 0$

   C. $= \dfrac{1}{5}$

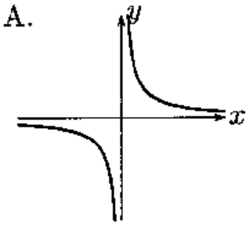
   D. $= -\infty$

   E. $= -\dfrac{4}{3}$

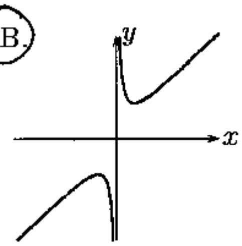
$$= \lim_{x\to\infty} \frac{\frac{2}{x^2} - \frac{3}{x} - 4}{\frac{10}{x^2} + \frac{6}{x} + 3} = -\frac{4}{3}$$

3. The graph of $f(x) = x + \dfrac{4}{x}$ looks most like which graph below?

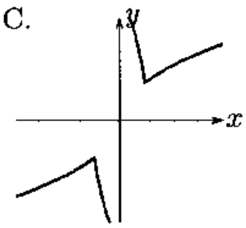
A.



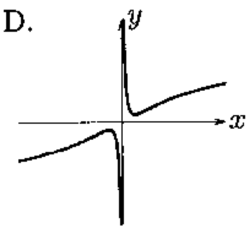
B.



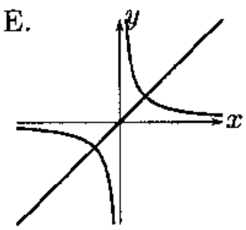
C.



D.



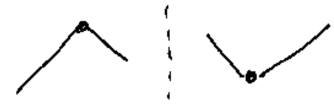
E.



$$f'(x) = 1 - \frac{4}{x^2} = \frac{x^2-4}{x^2}$$

	$-\infty$	-2		0		2	∞ x
x^2-4	$+$	0	$-$	¦	$-$	0	$+$

graph of f



vertical asymptote at $x=0$

$$f''(x) = \frac{8}{3} \begin{cases} >0 & \text{if } x>0 \\ <0 & \text{if } x<0 \end{cases} \Rightarrow$$

$\Rightarrow$ graph is B

4. The graph of $f(x) = 3x^5 - 5x^3$ is concave downward on the interval(s).

A. $(-\infty, -\frac{1}{2}) \cup (0, \frac{1}{2})$

B. $(-\frac{1}{2}, 0) \cup (\frac{1}{2}, \infty)$

C. $(-\infty, -\frac{\sqrt{2}}{2}) \cup (0, \frac{\sqrt{2}}{2})$

D. $(-\frac{\sqrt{2}}{2}, 0) \cup (\frac{1}{2}, \infty)$

E. none of the above

$f'(x) = 15x^4 - 15x^2$

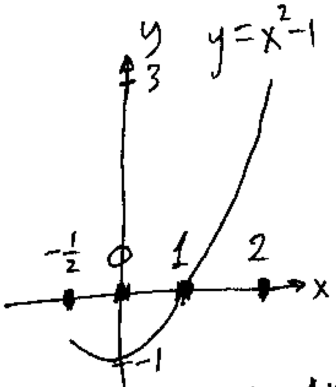
$f''(x) = 60x^3 - 30x$

$= 30x(2x^2 - 1)$

x: $-\infty$ to $+\infty$	$-\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{2}}$	
$30x$	$-$	$-$	$+$	$+$
$2x^2-1$	$+$	$-$	$-$	$+$
$f''(x)$	$-$	$+$	$-$	$+$

graph of f concave down down

5. If $f(x) = x^2 - 1$ and $P = \{-\frac{1}{2}, 0, 1, 2\}$, then the upper sum $U_f(P) =$



x:	$-\frac{1}{2}$	0	1	2
x^2-1:	$-\frac{3}{4}$	-1	0	3

A. $-\frac{11}{8}$

B. $-\frac{3}{8}$

C. 0

D. $\frac{20}{8}$

E. $\frac{21}{8}$

max $\cdot \Delta x$ + max $\cdot \Delta x$ + max $\cdot \Delta x$

$U_f(P) = \left(-\frac{3}{4}\right)\left(\frac{1}{2}\right) + (0)(1) + (3)(1)$

$= \frac{21}{8}$

6. The values of a and b which guarantee that $\int_a^b f(t)dt - \int_5^3 f(t)dt = \int_3^1 f(t)dt$ are

A. $a = 5,\ b = 1$
B. $a = 4,\ b = 2$
C. $a = 2,\ b = 4$
D. $a = 1,\ b = 2$
E. $a = 3,\ b = 1$

$$\Rightarrow \int_a^b f(t)dt = \int_5^3 f(t)dt + \int_3^1 f(t)dt$$
$$= \int_5^1 f(t)dt$$

$$\Rightarrow a=5, \quad b=1$$

7. If $f(x) = \begin{cases} 4, & 1 \le x \le 3 \\ 2x-2, & 3 < x \le 4 \end{cases}$, then $\int\limits_1^4 f(x)dx =$

A. 3
B. 8
C. 13
D. 18
E. 23

$$\int_1^4 f(x)\,dx = \int_1^3 f(x)\,dx + \int_3^4 f(x)\,dx$$
$$= \int_1^3 4\,dx + \int_3^4 (2x-2)\,dx$$
$$= 4x\Big|_1^3 + (x^2-2x)\Big|_3^4$$
$$= (12-4) + ((8) - (3))$$
$$= 8 + 5$$
$$= 13.$$

8. If $F(x) = \int_{2}^{x^4} \sin t^2 dt$, then $F'(a) =$

$F'(x) = (\sin(x^4)^2)(4x^3)$

$F'(a) = (\sin a^8)(4a^3)$

A. $a^4 \sin a^8$

B. $4a^3 \sin a^2$

C. $a^4 \cos a^2$

D. $4a^3 \cos a^2$

E. $4a^3 \sin a^8$

9. $\int_{1}^{2} (x + \frac{1}{x})^2 dx =$

$= \int_1^2 (x^2 + 2 + x^{-2})\, dx$

$= \left(\frac{x^3}{3} + 2x + \frac{x^{-1}}{-1}\right)\Big|_1^2$

$= \left(\frac{8}{3} + 4 - \frac{1}{2}\right) - \left(\frac{1}{3} + 2 - 1\right)$

$= \frac{29}{6}$

A. $\frac{31}{6}$

B. $\frac{29}{6}$

C. $\frac{23}{6}$

D. $\frac{20}{6}$

E. $\frac{17}{6}$

10. The function $F(x) = \int_0^x (\sqrt{t} - t^3)dt$ is increasing for

A. $0 < x < 1$

B. $x > 0$

C. $x > \sqrt[3]{2}$

D. $0 < x < \sqrt[3]{2}$

E. $x > 1$

$$F'(x) = \sqrt{x} - x^3 = 0.$$

$$\longrightarrow \sqrt{x}\left(1 - x^{5/2}\right) = 0$$

$$\longrightarrow x = 0, 1.$$

$0 \quad 1 \quad \infty \quad x$

$1 - x^{5/2}$ $+ \quad 0 \quad -$

$\Rightarrow F$ inc.   dec.

$\Rightarrow$ F increasing for $0 < x < 1$

11. $\int_0^{\frac{1}{2}} \frac{3x}{(x^2-1)^2}\, dx =$

A. $\frac{1}{2}$

B. $\frac{3}{4}$

C. 2

D. 3

E. 4

(Let $u = x^2 - 1$. Then $du = 2\,dx$.
$u(0) = -1$, $u(\frac{1}{2}) = -\frac{3}{4}$)

$$= \int_{-1}^{-3/4} u^{-2}\, 3\left(\tfrac{1}{2}du\right)$$

$$= \frac{3}{2} \cdot \frac{u^{-1}}{-1}\Big|_{-1}^{-3/4} = -\frac{3}{2}\left(-\frac{4}{3} - (-1)\right)$$

$$= 1/2$$