MATH 161 – 161E – SPRING 1998 – FIRST EXAM
February 17, 1998

STUDENT NAME Solution Key

STUDENT ID

RECITATION INSTRUCTOR

INSTRUCTIONS:

1. This test booklet has 6 pages including this page.
2. Fill in your name, your student ID number, and your recitation instructor's name above.
3. Use a number 2 pencil on the mark-sense sheet (answer sheet).
4. On the mark-sense sheet, fill in the recitation instructor's name and the course number.
5. Fill in your name and student ID number, blacken the appropriate spaces, and sign the mark-sense sheet.
6. Mark the the division and section number of your class and blacken the corresponding circles, including the circles for the zeros. If you do not know your division and section number ask your instructor.
7. There are 15 questions, each worth 7 points. Blacken your choice of the correct answer in the spaces provided for questions 1-15. Turn in BOTH the answer sheet and the question sheets to your instructor when you are finished.
8. No books, notes or calculators may be used.

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1) Solve the inequality $|5 - 3x| \leq 3$.

a) $-\frac{2}{3} \leq x \leq -\frac{8}{3}$

b) $x \leq -\frac{8}{3}$

c) $\frac{2}{3} \leq x \leq \frac{8}{3}$

d) $x \leq \frac{2}{3}$

e) all x

$|5-3x| \leq 3 \rightarrow -3 \leq 5-3x \leq 3$

$\rightarrow -8 \leq -3x \leq -2$

$\rightarrow \frac{8}{3} \geq x \geq \frac{2}{3}$

$\rightarrow \frac{2}{3} \leq x \leq \frac{8}{3}$

line ℓ:

2) An equation of the line perpendicular to $3y = x - 21$ and passing through $(3, -1)$ is

$\ell: \rightarrow y = \frac{1}{3}x - 7 \rightarrow$ has slope $\frac{1}{3}$

$\rightarrow$ perpendicular line has slope -3

Perp. line is: $y - (-1) = -3(x-3)$

$\rightarrow y + 1 = -3x + 9$

$\rightarrow y = -3x + 8$

a) $y = \frac{1}{3}x - 2$

b) $x = \frac{1}{3}y - 2$

c) $x = \frac{1}{3}x + \frac{10}{3}$

d) $y = -3x + 8$

e) $y = 3x - 10$

3) If $f(t) = \frac{2t}{t+1}$, then $\frac{f(t)-f(2)}{t-2}$ equals, for $t \neq 2$,

a) $\frac{2}{3(t+1)}$

b) $\frac{3}{2(t+1)}$

c) $\frac{2}{3(t-2)}$

d) $\frac{3}{2(t-2)}$

e) $\frac{4t-6(t+1)}{3(t-2)(t+1)}$

$$\frac{f(t)-f(2)}{t-2} = \frac{\frac{2t}{t+1} - \frac{4}{3}}{t-2} = \frac{3(2t) - (t+1)(4)}{(t-2)(t+1)(3)}$$

$$= \frac{6t - 4t - 4}{(t-2)(t+1)(3)} = \frac{2t-4}{(t-2)(t+1)(3)} = \frac{2(t-2)}{(t-2)(t+1)(3)}$$

$$= \frac{2}{(t+1)(3)} \text{ for } t \neq 2.$$

4) The graph of $\ln|y - 3| = (x + 2)^3 - 4(x + 2)$ can be obtained by moving the graph of $\ln|y| = x^3 - 4x$,

a) 2 units left and 3 units down

b) 3 units left and 2 units down

c) 2 units right and 3 units up

d) 2 units left and 3 units down

e) 2 units left and 3 units up

$x - h = x + 2 = x - (-2)$

$\Rightarrow$ graph moved 2 units left.

$y - k = y - 3$

$\Rightarrow$ graph moved 3 units up.

5) If $f(x) = \frac{3x^2}{x+1}$ and $g(x) = x^2$, then $g \circ f(-2)$ equals

a) $-2\left(\frac{3x^2}{x+1}\right)^2$

b) $\frac{-6x^4}{x^2+1}$

c) $\frac{48}{5}$

d) 144

e) $-2x^2\left(\frac{3x^2}{x+1}\right)$

$$g(f(-2)) = g\left(\frac{3(-2)^2}{-2+1}\right) = g(-12)$$

$$= (-12)^2 = 144.$$

6) Let

$$f(x) = \begin{cases} 3x - 1 \text{ for } x > 1 \\ ax^2 \text{ for } x \leq 1 \end{cases}$$

If f is continuous at $x = 1$, then a is equal to

a) 10

b) 2

c) 5

d) 4

e) 8

f cont. at $x=1$ means $\lim_{x \to 1} f(x) = f(1)$.

$$\left.\begin{array}{l} \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} ax^2 = a(1)^2 = a \\ \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 3x - 1 = 3(1) - 1 = 2 \end{array}\right\} \Rightarrow a = 2$$

$f(1) = a(1)^2 = a$, which is 2 by above. So $\lim_{x \to 1} f(x) = f(1)$.

7) The values of x for which

$$\ln(x-3) + \ln(x+2) - \ln(4x) = 0$$

are

a) $x > 3$

b) $x = 5$

c) $x = -1$ and $x = 6$

d) $x = 6$

e) No values of x

$\rightarrow \ln \frac{(x-3)(x+2)}{4x} = 0$

$\rightarrow \frac{(x-3)(x+2)}{4x} = e^0 = 1$

$\rightarrow (x-3)(x+2) = 4x$

$\rightarrow x^2 - x - 6 = 4x$

$\rightarrow x^2 - 5x - 6 = 0$

$\rightarrow (x-6)(x+1) = 0$

$\rightarrow x = 6, -1$ BUT $x = -1$ is not a solution of $\ln(x-3) + \ln(x+2) - \ln(4x) = 0$.

8)

$$\lim_{x \to 1} \frac{x^2 + 4x - 5}{x^2 - 1} \quad \text{is equal to}$$

(note: limit is $\frac{0}{0}$)

a) 0

b) 1

c) 2

d) 3

e) Not defined

$$= \lim_{x \to 1} \frac{(x+5)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}} = \frac{6}{2} = 3.$$

9) An equation of the tangent line to $f(x) = e^x \sin(x) + 1$ at $(0, f(0))$ is

a) $y = x + 1$

b) $y = 2x + 1$

c) $2y = x + 2$

d) $3y = 4x + 3$

e) $y = 5x + 1$

Derivative is slope : $f'(x) = e^x \sin x + e^x \cos x$.

$$f'(0) = e^0 \sin 0 + e^0 \cos 0 = e^0 \cdot 0 + e^0 \cdot 1 = e^0 = 1.$$

Also $f(0) = e^0 \sin 0 + 1 = 1$

tangent line: $y - 1 = (1)(x - 0)$

$\longrightarrow y = x + 1.$

10) The vertical asymptotes of

$$f(x) = \frac{x^2 - 4}{(x-4)(x^2 - 5x + 6)} \quad \text{are}$$

a) $x = 2$, $x = 4$ and $x = 3$

b) $x = 4$ and $x = 3$

c) $x = 2$, and $x = 3$

d) $x = 3$

e) $x = 2$

$$f(x) = \frac{\cancel{(x-2)}(x+2)}{(x-4)\cancel{(x-2)}(x-3)}, \quad x \neq 2.$$

Vertical asymptotes: $x - 4 = 0 \to x = 4$

$x - 3 = 0 \to x = 3$

11)

$$\lim_{x\to -1}\left(\frac{1}{x+1}+\frac{2}{x^2-1}\right) \quad = \quad -\infty + \infty$$

$(\text{or } \infty - \infty)$

is equal to

a) -1

b) $\frac{-1}{2}$

c) 0

d) $\frac{1}{2}$

e) 1

$$\frac{1}{x+1} + \frac{2}{x^2-1} = \frac{(x-1)+2}{x^2-1} = \frac{x+1}{x^2-1}$$

$$= \frac{x+1}{(x+1)(x-1)} = \frac{1}{x-1} \quad \text{if } x \neq -1$$

and $\lim\limits_{x\to -1} \frac{1}{x-1} = \frac{1}{-2}$.

12)

$$\lim_{x\to 4^-}\frac{|x-4|}{x^2-4x}$$

is equal to

a) $\frac{-1}{2}$

b) $\frac{-1}{4}$

c) 0

d) $\frac{1}{4}$

e) $\frac{1}{2}$

$x \to 4^- \Rightarrow x < 4 \Rightarrow x - 4 < 0$

So, $\lim\limits_{x\to 4^-} \frac{|x-4|}{x^2-4x} = \lim\limits_{x\to 4^-} \frac{-(x-4)}{(x)(x-4)}$

$= \lim\limits_{x\to 4^-} \frac{-1}{x} = \frac{-1}{4}$.

13) If $f(x) = |x|$, $g(x) = x|x|$ and $h(x) = \sin(x)$, which of these functions are differentiable at $x = 0$?

a) None

b) f and h

c) Only g

d) g and h

e) Only f

$f(x) = |x|$ has graph with corner at $(0,0)$.

$\Rightarrow f$ <u>not</u> differentiable at $x=0$.

$g(x) = x|x| = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$ and has graph that is smooth at $(0,0)$

$\Rightarrow g$ <u>is</u> differentiable at $x=0$.

$h(x) = \sin(x)$ <u>is</u> differentiable at $(0,0)$

14) The slope of the tangent line to the graph of

$$f(x) = 3xe^{4x}\cos(9x^2) + x^2e^{3x}$$

at $x = 0$ is

a) 5

b) 3

c) $e^4 \cos(36)$

d) $3\cos(4)$

e) 1

15) The derivative of $-\ln\left((\cos x)^6\right)$ is

a) $\tan x + 6$

b) $6 \cot x$

c) e^x

d) $6 \tan x$

e) $6x + 2.$

Note: Problems 14 and 15 require the chain rule which is not covered on MA 161/161E exam 1, Spring, 2000.