MA161 **FINAL EXAM** **December 15, 1997**

Name: _____

ID #: _____

Recitation Instructor _____ Time of Recitation _____

Section #: _____

Instructions:

1. Fill in your name, student ID number and division and section numbers on the mark–sense sheet. Also fill in the information requested above.
2. This booklet consists of 14 pages. There are 25 questions, each worth 8 points.
3. Mark your answers on the mark–sense sheet. Please show your working in this booklet.
4. No books, notes or calculators please.
5. When you are finished with the exam, hand this booklet and the mark–sense sheet, in person, to your instructor.
6. Have a nice holiday.

1. If $|2 - 8x| > 1/2$ then
   A. $3/16 < x < 5/16$
   B. $x < 3/16$ or $x > 5/16$
   C. $0 < x < 5/16$
   D. $5/16 < x < \infty$
   E. None of the above

2. $\lim_{x \to \pi/3} \ln(\ln(2 \sin x)) =$
   A. $\ln(\ln 3) - \ln 2$
   B. $2 \ln 3 - \ln 2$
   C. $\ln 2 + 2 \ln 3$
   D. $\ln(\ln 2) - \frac{1}{3} \ln 2$
   E. $\ln(\ln 2) - \ln 3$

3. Consider the tangent to the curve $y = x^3$ at $(2, 8)$. What is the equation of the line that is perpendicular to this tangent and passes through the point $(1, 5)$?

   A. $y - 12x + 7 = 0$

   B. $2y - 5x + 2 = 0$

   C. $y + x - 61 = 0$

   D. $12y + x - 61 = 0$

   E. None of the above

4. The function $f(x) = \begin{cases} x^2 + 1 & 0 \leq x \leq 3 \\ 10x/3 & 3 < x < \infty \end{cases}$ is

   A. continuous for all $x \geq 0$

   B. continuous for all $x \geq 0$ except at $x = 3$

   C. continuous only for $0 \leq x \leq 3$

   D. continuous only for $3 < x < \infty$

   E. None of the above is true

5. $\lim_{x \to -2} \dfrac{x^2 - 4}{|x| - 2}$

   A. Does not exist

   B. is -4

   C. is 4

   D. is 0

   E. is ∞

6. If $y = \dfrac{\sinh x}{x^2 + 1}$ then $\dfrac{dy}{dx}$ is

   A. $\dfrac{\cosh x}{2x}$

   B. $\dfrac{x^3 \cosh x - x^2 \sinh x}{(x^2 + 1)^2}$

   C. $\dfrac{(x^2 + 1) \cosh x - 2x \sinh x}{(x^2 + 1)^2}$

   D. $\dfrac{\cosh x - x \sinh x}{(x^2 + 1)^2}$

   E. None of the above

7. Suppose $1 - 2x^2 \leq g(x) \leq -8x + 9$ for $0 \leq x \leq 4$. Then $\lim_{x \to 2} g(x) =$

   A. 2

   B. -7

   C. -8

   D. -16

   E. There is not enough information to determine the limit.

8. A missile is launched vertically. After t seconds its altitude is $36t \ln(1 + t)$ meters above ground. What is its acceleration after 5 seconds?

   A. 7m/s^2

   B. 9m/s^2

   C. 5m/s^2

   D. 20m/s^2

   E. 25m/s^2

9. The slope of the tangent line to the curve $x^3 + y^3 + 2y = 4$ at the point $(1, 1)$ is

   A. 1

   B. $-2/5$

   C. $-3/5$

   D. $3/2$

   E. 2

10. Two sides of an isosceles triangle are 3 inches long, and the angle between them is increasing at the rate 1 rad/min. At the moment when the third side is also 3 inches long, at what rate is this side increasing?

    A. $\frac{3\sqrt{3}}{2}$ in/min

    B. 1 in/min

    C. $\frac{\sqrt{3}}{2}$ in/min

    D. $\frac{1}{2}$ in/min

    E. $\frac{\sqrt{3}}{3}$ in/min

11. The sum of two positive angles, α and β is $\pi/2$. What is the maximum value of $\sin\alpha + \sin\beta$?

    A. 1

    B. 3/2

    C. $\sqrt{2}$

    D. $\dfrac{\sqrt{2}}{2}$

    E. There is no maximum

12. A function h is continuous and differentiable on $(-\infty, \infty)$. We know $h(0) = 0$ and $h(1) = 2$. Which of the following must be true?

    I. On the interval $[0, 1]$ h has a maximum.

    II. There is an $x,\ 0 \le x \le 1$, such that $h'(x) = 0$.

    III. There is an $x,\ 0 \le x \le 1$, such that $h'(x) = 2$.

    A. Only I

    B. Only I and II

    C. Only I and III

    D. Only II and III

    E. All three

13. The relative extrema of the function $\ln(e^x + e^{-x})$ are as follows.

    A. Relative minimum at 0, relative maxima at $1/e$ and $-1/e$.

    B. Relative minimum at $1/e$ and $-1/e$, relative maximum at 0.

    C. There is no relative minimum, there is relative maximum at 0.

    D. Relative minimum at 0, but there is no relative maximum.

    E. There are no relative extrema.

14. $\lim_{x\to\infty} \dfrac{x - 1/x + \sin 1/x}{2x + \sqrt{1+x}} =$

    A. $-1/2$

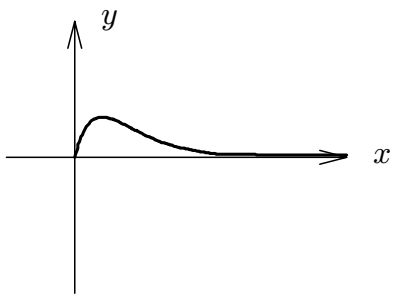
    B. 0

    C. $1/3$

    D. $1/2$

    E. ∞

(15.)



. This could be the graph of the function

A. $e^{-2x} - e^{-3x},\ x > 0$

B. $1/(1+x),\ x > 0$

C. $x/(1+x),\ x > 0$

D. $1/\ln x,\ x > 0$

E. $xe^x,\ x > 0$

16. $\int_{-1}^{2} |x^3| dx =$

A. 17/4

B. 15/4

C. 1/2

D. 13/4

E. 11/4

17. If $\int_{-2}^{2} f(x)dx = 0$, which of the following statements must be true?

    I. $f(x) = 0$ for all x in $[-2, 2]$

    II. $|f(x)| \geq 1$ for some x in $[-2, 2]$

    III. $\int_{0}^{-2} f(x)dx = \int_{0}^{2} f(x)dx$

    A. All three

    B. Only I and III

    C. Only I and II

    D. Only III

    E. None

18. The area enclosed by the curve $x = y^2$ and the line $y = x - 2$ is

    A. 7/2

    B. 9/2

    C. 11/2

    D. 13/2

    E. 15/2

19. $\dfrac{d}{dx}\displaystyle\int_2^{e^x} \frac{dt}{\ln t} =$

A. xe^x

B. xe^{-x}

C. e^{-x}/x

D. e^x/x

E. $1/x$

20. $\dfrac{d}{dx}(2x)^x =$

A. $x(2x)^{x-1}$

B. $(2x)^x \ln 2$

C. $(2x)^x / \ln 2$

D. $(2x)^x \ln(2x)$

E. $(2x)^x(1 + \ln(2x))$

21. $\int_0^2 4^x dx =$

A. $8/\ln 2$

B. $8\ln 2$

C. $\dfrac{15}{2\ln 2}$

D. 60

E. 15

22. $\int \dfrac{2x}{\sqrt{1-x^4}}\, dx =$

A. $\sin^{-1}(x^2) + C$

B. $\tan^{-1}(x^2) + C$

C. $\ln\sqrt{1-x^4} + C$

D. $\sqrt{1-x^4} + C$

E. $\dfrac{\sqrt{1-x^4}}{x^2} + C$

23. If $f(x) = x^5 + 4x$ then $(f^{-1})'(5)$ is

A. 1

B. 1/4

C. 1/5

D. 1/9

E. 1/20

24. $\int_0^{1/3} \frac{dx}{1+9x^2} =$

A. $\pi/18$

B. $\pi/12$

C. $\pi/6$

D. $\pi/3$

E. $\pi/2$

25. $\tan(\sin^{-1} x) =$

A. $\dfrac{x}{1+x^2}$

B. $\dfrac{1}{1+x^2}$

C. $x\sqrt{1-x^2}$

D. $\dfrac{1}{\sqrt{1-x^2}}$

E. $\dfrac{x}{\sqrt{1-x^2}}$