MA 162 Exam 2 Spring 1998

NAME _____

STUDENT ID _____

REC. INSTR. _____ REC. TIME. _____

INSTRUCTOR _____

INSTRUCTIONS:

1. Make sure that you have all 5 test pages.
2. Fill in your name, your student ID number, and your instructor's name above. Write your name, your student ID number and division and section number of your recitation section on your answer sheet, and fill in the corresponding circles.
3. Mark the letter of your response for each question on the mark–sense answer sheet.
4. There are 12 problems, each worth 8 points, for a total of 96 points.
5. No books or notes or calculators may be used.

Let R be the region between the graphs of f and g on $[a, b]$. Then the moments of R about x and y axes are

$$M_x = \int_a^b \frac{1}{2}(f(x)^2 - g(x)^2)dx$$

$$M_y = \int_a^b x(f(x) - g(x))dx.$$

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1. Find $\displaystyle\int_0^2 \frac{x}{4 - x^2}\, dx$.

   A. $\frac{1}{2} \ln 4$

   B. $\ln 4$

   C. $\frac{1}{2}$

   D. 1

   E. Diverges

2. Let D be the region bounded by $y = x - x^2$ and $y = 0$. What is the volume of the solid obtained by revolving D about the y-axis?

   A. $\frac{\pi}{6}$

   B. $\frac{\pi}{3}$

   C. $\frac{\pi}{2}$

   D. $\frac{2\pi}{3}$

   E. $\frac{5\pi}{6}$

3. Let C be the portion of the curve $y = \sin(\pi x)$ between the points $(0, 0)$ and $(\frac{1}{2}, 1)$. Which integral represents the arc length of C?

   A. $\displaystyle\int_0^1 \sqrt{1 + \pi^2 \cos^2(\pi x)}\, dx$

   B. $\displaystyle\int_0^{\frac{1}{2}} \sqrt{1 + \cos^2(\pi x)}\, dx$

   C. $\displaystyle\int_0^1 \sqrt{1 + \pi^2 \sin^2(\pi x)}\, dx$

   D. $\displaystyle\int_0^{\frac{1}{2}} \sqrt{1 + \pi^2 \cos^2(\pi x)}\, dx$

   E. $\displaystyle\int_0^{\frac{\pi}{2}} \sqrt{1 + \pi^2 \sin^2(\pi x)}\, dx$

4. A conical tank with a diameter of 4 feet and a height of 8 feet is filled with water. Which integral represents the work required to pump out the water down to a depth of 4 feet?

   A. $\displaystyle\int_0^4 (62.5)(4 - y)\frac{\pi y^2}{16}\, dy$

   B. $\displaystyle\int_4^8 (62.5)(8 - y)\frac{\pi y^2}{4}\, dy$

   C. $\displaystyle\int_4^8 (62.5)(8 - y)\frac{\pi y^2}{16}\, dy$

   D. $\displaystyle\int_0^4 (62.5)(4 - y)\frac{\pi y^2}{4}\, dy$

   E. $\displaystyle\int_4^8 (62.5)(8 - y)16\pi\, dy$

5. If it takes a force of 1 pound to stretch a spring 2 feet beyond its natural length, how much work is required to stretch it an additional two feet?

   A. 3 ft-lbs

   B. 12 ft-lbs

   C. 4 ft-lbs

   D. 16 ft-lbs

   E. 2 ft-lbs

6. Let D be the region bounded by $y = 3$, $x = 0$, and $y = \frac{3}{2}x$. Find the center of gravity of D.

A. $(1, \frac{3}{2})$
B. $(\frac{2}{3}, 2)$
C. $(2, 6)$
D. $(\frac{1}{3}, 1)$
E. $(2, \frac{2}{3})$

7. The third Taylor polynomial about 0 of $f(x) = x \sin x$ is

A. $x + \dfrac{x^2}{2}$
B. $\dfrac{x^2}{2} - x^3$
C. x^2
D. $1 + x^2$
E. $1 + x + \frac{x^2}{2}$

8. $\lim\limits_{n\to\infty} \dfrac{5^n}{(-6)^{n+2}} =$

A. 0
B. $\frac{5}{6}$
C. $-\frac{5}{6^2}$
D. ∞
E. limit does not exist

9. The series $\displaystyle\sum_{n=0}^{\infty} \frac{\sqrt{n}}{n^{\frac{3}{2}}+1}$

A. converges because $\dfrac{\sqrt{n}}{n^{\frac{3}{2}}+1} \to 0$ as $n \to \infty$

B. converges by the Comparison Test with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}}$

C. diverges by the Ratio Test

D. diverges by the Comparison Test with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

E. converges by the Integral Test

10. $\displaystyle\sum_{n=0}^{\infty} \frac{8^n - 9^n}{12^n} =$

A. -1

B. 1

C. $\frac{12}{11}$

D. $-\frac{12}{11}$

E. 2

11. Consider the series $S_1 = \sum_{n=0}^{\infty} \frac{n}{2^n}$ and $S_2 = \sum_{n=0}^{\infty} \frac{1}{(n+1)^2}$

A. S_1 diverges and S_2 converges

B. S_1 diverges and S_2 diverges

C. S_1 converges and S_2 diverges

D. S_1 converges and S_2 converges

E. $S_1 + S_2$ diverges

12. If $a_n = \frac{n!}{n^n}$, then $\sum_{n=1}^{\infty} a_n^{\frac{1}{2}}$

A. converges by the Ratio Test

B. converges because $a_n^{\frac{1}{2}} \to 0$ as $n \to \infty$

C. diverges because $n! \to \infty$ as $n \to \infty$

D. converges because $n^n \to \infty$ as $n \to \infty$

E. diverges because $\{a_n^{\frac{1}{2}}\}$ does not converge to 0 as $n \to \infty$