NAME ______________________________

STUDENT ID ______________________________

REC. INSTR. ______________ REC. TIME ______________

INSTRUCTIONS:

1. Verify that you have all the pages (there are 11 pages).
2. Fill in your name, your student ID number, and your recitation instructor's name and recitation time above. Write your name, your student ID number and division and section number of your recitation section on your answer sheet, and fill in the corresponding circles.
3. Mark the letter of your response for each question on the mark–sense answer sheet.
4. There are 25 problems. All problems are worth 8 points each.
5. No books or notes or calculators may be used.

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!},\ |x| < \infty \qquad \sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1},\ |x| < \infty$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n},\ |x| < \infty \qquad (1+x)^s = \sum_{n=0}^{\infty} \binom{s}{n} x^n,\ |x| < 1$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n},\ |x| < 1 \qquad \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n,\ |x| < 1$$

$$\sin^2 x = \frac{1-\cos(2x)}{2} \qquad \cos^2 x = \frac{1+\cos(2x)}{2}$$

$$\sin(2x) = 2\sin x \cos x \qquad 1+ = \tan^2 x = \sec^2 x$$

The angle of rotation θ, $0 < \theta < \pi/2$, that eliminates the xy term from the second degree equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ satisfies the equation $\tan 2\theta = \dfrac{B}{A-C}$, provided $A \neq C$. If $A = C$, then $\theta = \pi/4$.

$x = (\cos\theta)X - (\sin\theta)Y$ and $y = (\sin\theta)X + (\cos\theta)Y$, where the XY coordinate system is obtained by rotation the x and y axes through the angle θ about the origin.

$$\text{Arc length } = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta.$$

1. Find a vector of length one that is perpendicular to $\mathbf{v} = \mathbf{i} - \mathbf{j}$ and $\mathbf{w} = 2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$.

A. $\left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$

B. $(2, -1, 1)$

C. $\left(\frac{2}{9}, \frac{2}{9}, \frac{1}{9}\right)$

D. $\left(\frac{2}{9}, -\frac{2}{9}, \frac{1}{9}\right)$

E. $(2, 2, 1)$

2. A person pulls a sled 15 feet with a rope that makes an angle of 30° with the horizontal ground. Find the work done if the tension in the rope is 20 pounds.

A. 300 ft-lbs

B. 150 ft-lbs

C. $150\sqrt{3}$ ft-lbs

D. $300\sqrt{3}$ ft-lbs

E. $75\sqrt{3}$ ft-lbs

3. Evaluate $\lim_{x \to 0^+} (1 + x^2)^{\frac{3}{x}}$.

A. ∞

B. e^3

C. $e^{\frac{3}{2}}$

D. e

E. 1

4. Compute $\int_0^1 2x \sin \pi x \, dx$.

A. 2

B. $\frac{2}{\pi}$

C. $\frac{2}{\pi} - \frac{2}{\pi^2}$

D. $\frac{2}{\pi^2}$

E. $\frac{4}{\pi} - \frac{4}{\pi^2}$

5. Compute $\int_0^1 \frac{dx}{x^2 + 3x + 2}$.

A. $2 \ln 2 - \ln 3$

B. $\ln 2 - \frac{1}{2} \ln 3$

C. $\ln 2 - \ln 3$

D. $\ln 6 - \ln 2$

E. $\ln 6$

6. In order to compute $\int \frac{dx}{\sqrt{4 + 9x^2}}$ one makes a trigonometric substitution of the form $x = a \tan \theta$ for some value of a. What should a be and what is the resulting integral in θ?

A. $a = \frac{1}{3}, \int \frac{\sec \theta}{6} \, d\theta$

B. $a = \frac{1}{3}, \int \frac{\cos \theta}{2} \, d\theta$

C. $a = \frac{2}{3}, \int \frac{\cos \theta}{2} \, d\theta$

D. $a = \frac{2}{3}, \int \frac{\sec \theta}{3} \, d\theta$

E. $a = \frac{3}{2}, \int \frac{3}{4} \tan \theta \, d\theta$

7. Compute $\int \sin^2 x \cos^2 x dx$.

A. $\frac{x}{8} - \frac{1}{32} \sin 4x + C$

B. $\frac{x}{8} + \frac{1}{32} \sin 4x + C$

C. $\frac{x}{4} - \frac{1}{16} \sin 4x + C$

D. $\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C$

E. $\sin^3 x - \sin^5 x + C$

8. Evaluate the improper integral
$\int_1^\infty \frac{x}{(1+x^2)^{3/2}}\, dx$

A. $\frac{3}{\sqrt{2}}$

B. $\frac{1}{3\sqrt{2}}$

C. $\frac{1}{\sqrt{2}}$

D. $\frac{1}{2\sqrt{2}}$

E. diverges

9. The length of the curve $y = \frac{1}{3} x^{\frac{3}{2}} - 1$, $0 \le x \le 5$ is

A. 6

B. $\frac{\sqrt{8}}{3}$

C. $\frac{\sqrt{5}}{3} - 1$

D. $\frac{\sqrt{125}}{3}$

E. $\frac{19}{3}$

10. Find the volume of the solid obtained by revolving about the x axis the region in the first quadrant bounded by the curve $x^2 + 2y^2 = 2$.

A. 2π

B. $\frac{2\sqrt{2}}{3}\pi$

C. $\sqrt{2}\pi$

D. $\frac{5}{6}\pi$

E. $\frac{8}{3}\pi$

11. If it takes 4 lbs of force to stretch a spring from its natural length by 1 ft. how much work will it take (in ft-lbs) to extend it by an additional 1 ft?

A. 6

B. 8

C. 10

D. 12

E. 16

12. Two objects of equal weight are placed at $(-1, 0)$ and $(1, -1)$. Where should a third object which is twice as heavy be placed so that the center of mass of the 3 objects is at $(0, 0)$?

A. $(0, \frac{1}{2})$

B. $(2, -1)$

C. $(0, 1)$

D. $(1, 0)$

E. $(1, \frac{1}{2})$

13. $\displaystyle \lim_{k\to\infty} \frac{k^2 + 2\ln k}{2^k + k\ln 2} =$

A. 0

B. 1

C. $\ln 2$

D. ∞

E. limit does not exist

14. $\displaystyle \lim_{n\to\infty} \sqrt[n]{2n+1} =$

A. ∞

B. 1

C. 2

D. $\frac{1}{2}$

E. limit does not exist

15. $\displaystyle \sum_{n=0}^{\infty} (-1)^n \frac{3^n}{4^{n+1}} =$

A. $\dfrac{1}{7}$

B. $\dfrac{4}{7}$

C. 1

D. 4

E. $\dfrac{1}{5}$

16. Consider the following infinite series:

I. $\displaystyle\sum_{n=0}^{\infty} \frac{n}{\sqrt{n+2}}$, II. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2(\ln n)^2}$

Then

A. I and II both converge

B. I converges, II diverges

C. I and II both diverge

D. I diverges, II converges

E. I converges because the general term goes to 0

17. Consider the three infinite series:

I. $\displaystyle\sum_{n=1}^{\infty} n\left(\frac{2}{3}\right)^n$, II. $\displaystyle\sum_{n=1}^{\infty} \frac{(n-1)!}{(n+1)!}$ III. $\displaystyle\sum_{n=1}^{\infty} \frac{2^{n^2}}{n!}$. Then

A. I, II, III all converge

B. I, II converge, III diverges

C. I converges, II, III diverge

D. Only I and III converge

E. II converges, I, III diverge

18. The interval of convergence of $\displaystyle\sum_{n=1}^{\infty} n2^n x^n$ is

A. $[-\frac{1}{2}, \frac{1}{2})$

B. $[-2, 2]$

C. $[-1, 1]$

D. $(-2, 2)$

E. $(-\frac{1}{2}, \frac{1}{2})$

19. The first three terms of the Taylor series of $f(x) = \ln(2 + \sin x)$ about $x = 0$ are

A. $1 + \frac{x}{2} - \frac{x^2}{4}$

B. $\ln 2 - \frac{1}{2}x + \frac{x^2}{8}$

C. $\ln 2 + \frac{1}{2}x - \frac{x^2}{8}$

D. $\ln 2 + \frac{1}{2}x + \frac{x^2}{4}$

E. $1 + \frac{x}{2} - \frac{x^2}{8}$

20. $\int_0^{\frac{1}{2}} (1 + t^2)^{\frac{1}{3}}\, dt =$

A. $\dfrac{3}{4}$

B. $\left(\dfrac{5}{4}\right)^{\frac{4}{3}}$

C. $\displaystyle\sum_{n=0}^{\infty} \binom{\frac{1}{3}}{n} \frac{1}{(2n+1)2^{2n+1}}$

D. $\displaystyle\sum_{n=0}^{\infty} \binom{\frac{1}{3}}{n} \frac{1}{(n+1)2^{n+1}}$

E. $\tan^{-1}\left(\dfrac{1}{2}\right)$

21. The curve defined by $x = t - 1$, $y = t^2 + 2$, $\quad -1 \leq t \leq 1$ most closely resembles

A

B.

C.

D.

E.

22. The area of the region common to the circles $r = \sqrt{3}\cos\theta$ and $r = \sin\theta$ is represented by

A. $\int_0^{\frac{\pi}{6}} 3\cos^2\theta d\theta + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin^2\theta d\theta$

B. $\int_0^{\frac{\pi}{6}} \frac{3}{2}\cos^2\theta d\theta + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{1}{2}\sin^2\theta d\theta$

C. $\int_0^{\frac{\pi}{3}} \sin^2\theta d\theta + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 3\cos^2\theta d\theta$

D. $\int_0^{\frac{\pi}{3}} \frac{1}{2}\sin^2\theta d\theta + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{3}{2}\cos^2\theta d\theta$

E. $\int_0^{\frac{\pi}{3}} \frac{3}{2}\cos^2\theta d\theta + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{2}\sin^2\theta d\theta$

23. A given conic section is the collection of points whose distance from $(1, 3)$ is the same as the distance from the line $y = 1$. The equation of this conic section is:

A. $(x - 1)^2 = 4(y - 1)$

B. $(x - 1)^2 = 4(y - 3)$

C. $(x - 1)^2 + 4(y - 1)^2 = 1$

D. $4(x - 1)^2 + (y - 1)^2 = 1$

E. $(x - 1)^2 = 4(y - 2)$

24. Find an equation of the conic section whose vertices are $(5, 0)$, $(-5, 0)$ and the eccentricity is $\frac{4}{5}$.

A. $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$

B. $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$

C. $\dfrac{x^2}{25} - \dfrac{y^2}{9} = 1$

D. $\dfrac{x^2}{25} - \dfrac{y^2}{16} = 1$

E. $\dfrac{x^2}{16} + \dfrac{y^2}{25} = 1$

25. After a rotation of axes through an angle $\theta = \frac{1}{2}\tan^{-1}\left(\frac{24}{7}\right)$ the equation $2x^2 - 72xy + 23y^2 = 50$ in the xy coordinate system becomes $-25X^2 + 50Y^2 = 50$ in the XY coordinate system. Which of the following looks most like a graph of this equation?