NAME _____

STUDENT ID _____

REC. INSTR. _____ REC. TIME _____

INSTRUCTIONS:

1. Verify that you have all the pages (there are 5 pages).
2. Fill in your name, your student ID number, and your recitation instructor's name and recitation time above. Write your name, your student ID number and division and section number of your recitation section on your answer sheet, and fill in the corresponding circles.
3. Mark the letter of your response for each question on the mark–sense answer sheet.
4. There are 12 problems worth 8 points each.
5. No books or notes or calculators may be used.

Let R be the region between the graphs of f and g on $[a, b]$. Then the moments of R about x and y axes are

$$M_x = \int_a^b \frac{1}{2}(f(x)^2 - g(x)^2)dx$$

$$M_y = \int_a^b x(f(x) - g(x))dx.$$

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1. The region between the graphs of the functions $y = x^2$ and $y = 3x$ is revolved about the y axis. The volume of the solid so generated is

   A. $\frac{162}{5}\pi$

   B. $\frac{27\pi}{2}$

   C. 54π

   D. 48π

   E. 32π

2. The force required to hold a spring 1 inch beyond its natural length is 10 lbs. Find the work done to stretch it from 2 in. to 3 in. beyond natural length.

   A. 10 ft-lbs.

   B. $\frac{25}{12}$ ft-lbs.

   C. 1 ft-lbs.

   D. 2 ft-lbs.

   E. 5 ft-lbs.

3. A hemispherical tank with radius 12 feet is filled with a liquid of density δ. Find the amount of work W required to pump all the liquid in the tank to 4 feet above the top of the tank.

A. $\int_{-12}^{4} \delta(4-y)144\pi dy$

B. $\int_{-12}^{0} \delta(4-y)144\pi dy$

C. $\int_{-12}^{0} \delta(4-y)\pi(12-y)^2 dy$

D. $\int_{-12}^{0} \delta(4-y)\pi y^2 dy$

E. $\int_{-12}^{0} \delta(4-y)\pi(144-y^2) dy$

4. Find the length of the curve $y = f(x)$, $1 \leq x \leq 2$, if $f'(x) = \sqrt{x^4 - 1}$.

A. $\frac{7}{3}$

B. $\frac{31}{5}$

C. $\sqrt{7}$

D. 1

E. 2

5. Find the center of gravity of the region bounded by $y = x^2$, $y = 0$, $x = 1$.

A. $\left(\frac{2}{3}, \frac{1}{3}\right)$

B. $\left(\frac{3}{4}, \frac{1}{3}\right)$

C. $\left(\frac{1}{3}, \frac{1}{3}\right)$

D. $\left(\frac{3}{4}, \frac{3}{10}\right)$

E. $\left(\frac{1}{3}, \frac{3}{10}\right)$

6. Find the third Taylor polynomial, $P_3(x)$, for the function $f(x) = \tan x$.

A. $x - \frac{x^3}{3!}$

B. $x + \frac{x^3}{3!}$

C. $x + \frac{x^3}{3}$

D. $x + \frac{x^2}{2} + \frac{x^3}{3}$

E. $x - \frac{x^3}{3}$

7. $\lim_{n\to\infty} \sqrt{n}(\sqrt{n+1} - \sqrt{n-1}) =$

A. 0

B. 1

C. 2

D. 3

E. ∞

8. $\displaystyle\sum_{n=0}^{\infty} \frac{2^{2n} - 3^{n+1}}{5^n} =$

A. $-\frac{5}{2}$

B. $-\frac{1}{2}$

C. $\frac{3}{5}$

D. $\frac{25}{2}$

E. ∞

9. The series $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$

A. converges because $\frac{n^2}{n^3+1} \to 0$ as $n \to \infty$

B. converges by Comparison Test with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

C. diverges by the Root Test

D. diverges by the Ratio Test

E. diverges by the Limit Comparison Test with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

10. $\displaystyle\lim_{n\to\infty} \left(1 + \frac{e}{n}\right)^n =$

A. 1

B. $e^{\frac{1}{e}}$

C. e

D. e^e

E. ∞

11. Consider the series $S_1 = \sum_{n=0}^{\infty} \frac{n^{10}}{2^n}$ and $S_2 = \sum_{n=0}^{\infty} \frac{1}{2n^2+1}$

A. S_1 diverges and S_2 converges

B. S_1 diverges and S_2 diverges

C. S_1 converges and S_2 diverges

D. S_1 converges and S_2 converges

E. None of the above

12. The series $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^p}$

A. converges when $p < 0$

B. converges absolutely when $p \geq -1$

C. diverges when $p \geq 0$

D. diverges when $p \leq 1$

E. converges when $p > 0$