NAME ______________________________

STUDENT ID ______________________________

REC. INSTR. ______________ REC. TIME ______________

SECTION NUMBER ______________ LECTURER ______________

INSTRUCTIONS:

1. This package contains 13 problems, each worth 8 points.
2. Fill in the information requested above and on the mark-sense sheet.
3. Mark your answers on the mark-sense sheet and show work in this booklet.
4. No books or notes or calculators may be used.

1. $\lim_{k\to\infty} \sqrt[k]{k^2} =$

A. 0

B. 1

C. e

D. e^2

E. ∞

2. For fixed c $\lim_{n\to\infty} c^n = 0$ holds whenever

A. $c > 1$

B. $c \geq 1$

C. $-1 \leq c \leq 1$

D. $-1 \leq c < 1$

E. $-1 < c < 1$

3. $\lim_{n\to\infty} \frac{1+\sqrt{n}+n}{3-\sqrt{n}+2n} =$

A. -1

B. 0

C. $\frac{1}{3}$

D. $\frac{1}{2}$

E. the sequence diverges

4. $\sum_{m=0}^{\infty} \frac{2^{m+1}}{3^m} =$

A. 2

B. 3

C. 6

D. 12

E. ∞

5. Consider the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{\sqrt{n}+1}$. Which of the following statements is true?

A. The series diverges because $\lim_{n\to\infty} \frac{n}{\sqrt{n}+1} \neq 0$

B. The series can be seen to converge by the root test

C. The series can be seen to converge by comparison with $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

D. The series can be seen to converge by the alternating series test

E. The series can be seen to diverge by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$

6. The values of p for which the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ *converges absolutely* are

A. $p > 0$

B. $p \geq 1$

C. $0 < p < 1$

D. $0 < p \leq 1$

E. $p > 1$

7. Consider the following two statements.

I. $\sum_{n=1}^{\infty} \frac{1}{n2^n}$ can be seen to converge by comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

II. $\sum_{n=1}^{\infty} \frac{e^n}{n!}$ can be seen to converge by the ratio test.

A. Both I and II are true.

B. I is true, II is false.

C. I is false, II is true.

D. Both I and II are false

E. none of the above.

8. Apply the root test to $\sum_{n=1}^{\infty} \frac{\pi^n}{n3^{n+2}}$. The test indicates

A. The series converges absolutely

B. The series converges conditionally

C. The series diverges

D. The test is inconclusive

E. none of the above

9. The series $\sum_{n=1}^{\infty}(-1)^n \sin\frac{1}{n}$

A. is convergent and absolutely convergent

B. is convergent but not absolutely convergent

C. is absolutely convergent but not convergent

D. is not convergent nor is it absolutely convergent

E. None of the above

10. The series $\sum_{n=1}^{\infty}\frac{n}{n+1}$

A. converges because $\lim_{n\to\infty}\frac{n}{n+1}=1$

B. converges by the integral test

C. diverges because $\frac{n}{n+1}$ is decreasing

D. diverges because $\lim\left(\frac{n}{n+1}\right)^{1/n}=1$

E. diverges because $\lim_{n\to\infty}\frac{n}{n+1}=1$

11. The radius of convergence of $\sum_{n=0}^{\infty} x^{2n}$ is

A. 0

B. 1

C. $\sqrt{2}$

D. 2

E. ∞

12. The interval of convergence of $\sum_{k=0}^{\infty} \frac{x^k}{(2k)!}$ is

A. $[0, 1)$

B. $(-1, 1)$

C. $[-1, 1]$

D. $[0, \infty)$

E. $(-\infty, \infty)$

13. Given that $\sum_{k=0}^{\infty} x^{3k} = \dfrac{1}{1-x^3}$ if $|x| < 1$, find $\sum_{k=1}^{\infty} (3k) x^{3k-1}$

A. $\dfrac{3}{1-x^3}$

B. $\dfrac{3}{x(1-x^3)}$

C. $\dfrac{3x^2}{(1-x^3)^2}$

D. $\dfrac{1}{1-3x^2}$

E. $\dfrac{1}{(1-x^3)^3}$