NAME ______________________________

STUDENT ID ______________________________

REC. INSTR. ____________________ REC. TIME ____________________

INSTRUCTIONS:

1. Supply the information requested above, and on the mark–sense answer sheet.
2. Mark the letter of your response for each question on the mark–sense answer sheet; show your work in this booklet.
3. There are 25 problems; each worth 8 points.
4. No books, notes, or calculators, please. You may use the formulas supplied below though.
5. Have a good summer!

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!},\ |x| < \infty \qquad \sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1},\ |x| < \infty$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n},\ |x| < \infty \qquad (1+x)^s = \sum_{n=0}^{\infty} \binom{s}{n} x^n,\ |x| < 1$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n},\ |x| < 1 \qquad \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n,\ |x| < 1$$

$$\sin^2 x = \frac{1-\cos(2x)}{2} \qquad \cos^2 x = \frac{1+\cos(2x)}{2}$$

$$\sin(2x) = 2\sin x \cos x \qquad 1 + \tan^2 x = \sec^2 x$$

The angle of rotation θ, $0 < \theta < \pi/2$, that eliminates the xy term from the second degree equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ satisfies the equation $\tan 2\theta = \frac{B}{A-C}$, provided $A \neq C$. If $A = C$, then $\theta = \pi/4$.

$x = (\cos\theta)X - (\sin\theta)Y$ and $y = (\sin\theta)X + (\cos\theta)Y$, where the XY coordinate system is obtained by rotation the x and y axes through the angle θ about the origin.

$$\text{Arc length } = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta.$$

1. If $\mathbf{a} = 3\mathbf{i} + \mathbf{j}$, $\mathbf{b} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$, $\mathbf{c} = -3\mathbf{i} + \mathbf{j} + \mathbf{k}$ then

   A. $\mathbf{a}, \mathbf{b}$ and $\mathbf{b}, \mathbf{c}$ are perpendicular

   B. $\mathbf{a}, \mathbf{c}$ and $\mathbf{b}, \mathbf{c}$ are perpendicular

   C. $\mathbf{a}, \mathbf{c}$ are not perpendicular but $\mathbf{b}, \mathbf{c}$ are

   D. $\mathbf{a}, \mathbf{c}$ are perpendicular but $\mathbf{a}, \mathbf{b}$ are not

   E. None of the above

2. The area of the triangle with vertices at $P = (1, -1, 2)$, $Q = (2, 0, 1)$, and $R = (1, 2, -3)$ is

   A. 3

   B. $\sqrt{19/2}$

   C. $\sqrt{10}$

   D. $\sqrt{21/1}$

   E. $\sqrt{11}$

3. $\lim_{x\to\infty}\left(1+\frac{e}{x}\right)^{x/2} =$

A. 1

B. $\sqrt{e}$

C. $\sqrt{e^e}$

D. $e/2$

E. ∞

4. $\lim_{x\to 0}\frac{1-\cos\pi x}{1-\cos x} =$

A. 0

B. 1

C. π

D. π^2

E. ∞

5. $\int_{1}^{2} x \ln x dx =$

A. $\ln 2 + 1$

B. $\ln 2 - 1$

C. $\frac{1}{2}(\ln 2)^2$

D. $4 \ln 2 + \frac{3}{2}$

E. $2 \ln 2 - \frac{3}{4}$

6. The integral $\int \frac{1-x}{x^2(x+1)}\, dx$ will be of which of the following forms?

A. $\frac{a}{x} + b \ln |x| + c \ln |x+1| + d$

B. $a \ln |x| + b(\ln |x|)^2 + c \ln |x+1| + d$

C. $a \ln |x^2| + b \ln |x+1| + c$

D. $a \ln |x^2(x+1)| + b$

E. $\frac{a}{x} + \frac{b}{x+1} + \frac{c}{(x+1)^2} + d$

7. A suitable trigonometric substitution will transform the integral $\int \frac{dx}{(1+x^2)^{3/2}}$ into

A. $\int \cos\theta d\theta$

B. $\int \cos^2\theta d\theta$

C. $\int \sec^2\theta d\theta$

D. $\int \frac{d\theta}{\sec^3\theta}$

E. $\int (1+\theta^2)d\theta$

8. The improper integral $\int_0^1 \frac{dx}{x^a}$ converges when

A. $1 \leq a$

B. $a < 1$

C. $0 < a \leq 1$

D. $1 < a$

E. $0 < a$

9. The region under the curve $y = \dfrac{2}{\sqrt{1+x^2}}$, $0 \le x \le 1$, is rotated around the x axis. The volume of the solid of revolution is

A. $1/\pi^2$

B. $1/\pi$

C. 1

D. π

E. π^2

10. If $f'(x) = \sqrt{x^2 - 1}$ then the length of the curve $y = f(x)$, $2 \le x \le 3$ is

A. $5/2$

B. 3

C. $7/2$

D. 4

E. $9/2$

11. $\lim_{n\to\infty} \sqrt{n^2+2n} - \sqrt{n^2-2n} =$

A. 0

B. 1

C. 2

D. 4

E. ∞

12. Which of the following statements is true? The series $\sum_{n=1}^{\infty} \frac{1}{n+2^n}$ can be seen to

A. converge by the comparison test with $\sum_{n=1}^{\infty} \frac{1}{n}$

B. diverge by the comparison test with $\sum_{n=1}^{\infty} \frac{1}{n}$

C. converge by the comparison test with $\sum_{n=1}^{\infty} \frac{1}{2^n}$

D. diverge by the comparison test with $\sum_{n=1}^{\infty} \frac{1}{2^n}$

E. None of the above.

13. The generalized root test shows that the series $\sum_{n=1}^{\infty} \frac{(-n)^5}{5^n}$

A. converges absolutely
B. converges conditionally
C. diverges
D. test is inconclusive
E. none of the above

14. The series $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{2k-1}}$ is

A. convergent and absolutely convergent
B. convergent but not absolutely convergent
C. absolutely convergent but not convergent
D. neither convergent nor absolutely convergent
E. None of the above

15. The radius of convergence of the series $\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$ is

A. 0

B. $\frac{1}{2}$

C. 1

D. 2

E. ∞

16. The interval of convergence of the series $\sum_{n=1}^{\infty} \frac{x^{2n}}{n}$ is

A. $[0, 0]$

B. $[0, 1)$

C. $(-1, 1)$

D. $(-1, 1]$

E. $(-\infty, \infty)$

17. Given that the Taylor series of $\ln(1+x)$ about 0 is $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^n}{n}$, the Taylor series of $\ln(1-2x)$ is

A. $-\sum_{n=1}^{\infty} \frac{2^n x^n}{n}$

B. $-2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^n}{n}$

C. $\sum_{n=1}^{\infty} \frac{x^{2n}}{n}$

D. $2\sum_{n=1}^{\infty} \frac{x^n}{n}$

E. none of the above

18. In the Taylor series of $\tan x$ about $\pi/4$ the first 3 terms are

A. $1 + \left(x - \frac{\pi}{4}\right) + \frac{1}{2}\left(x - \frac{\pi}{2}\right)^2$

B. $1 + 2\left(x - \frac{\pi}{4}\right) + 2\left(x - \frac{\pi}{4}\right)^2$

C. $x - \frac{x^3}{6} + \frac{x^5}{120}$

D. $\frac{\pi}{4} + \frac{1}{\cos^2 x}\, x + \frac{\sin x}{\cos^3 x}\, x^2$

E. None of the above.

19. The Taylor series of $\dfrac{1}{\sqrt{1-x^4}}$ about 0 is

A. $1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} + \ldots$

B. $x + \dfrac{4x^5}{5} - \dfrac{4x^9}{25} + \ldots$

C. $x + x^5 + x^9 + \ldots$

D. $1 + \dfrac{x^4}{4} + \dfrac{5x^8}{18} + \ldots$

E. $1 + \dfrac{x^4}{2} + \dfrac{3x^8}{8} + \ldots$

20. The curve described parametrically by the equation $x = \cos^2 2t,\ y = \sin^2 2t$ looks most like

A.

B.

C.

D.

E.

21. At moment t an object is at the point $(x, y) = (\cos^3 t,\ \sin^3 t)$. Its (tangential) velocity when $t = \pi/4$ is

A. $1/2$

B. $\frac{\sqrt{2}}{2}$

C. 1

D. $\frac{3}{2}$

E. $\frac{\sqrt{3}}{2}$

22. The point with polar coordinates $r = 2,\ \theta = 3\pi$ has Cartesian coordinates

A. $(-2, 0)$

B. $(2, 3)$

C. $(1, 1)$

D. $(\sqrt{2}, \sqrt{2})$

E. $(\sqrt{3}, 1)$

23. The part of the first quadrant enclosed by the curve $r = \sqrt{\sin 3\theta}$ has area

A. $1/2$

B. $1/3$

C. π

D. $\pi/2$

E. $\pi/3$

24. The curve $2x + y^2 + 6y + 3 = 0$ looks most like

A.

B.

C.

D.

E.

25. Which of the following three statements is/are true? The equation
$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

I. can describe all parabolas, ellipses, and hyperbolas
II. can describe parabolas, ellipses, and hyperbolas only if $B = 0$
III. describes a parabola whenever $A = 0$

A. only I
B. only II
C. only III
D. all three
E. only I and III