SOLUTIONS TO PRACTICE QUESTIONS FOR THE FINAL EXAM

1. $16x^2 - 4y^8 = 4(4x^2 - y^8) = 4\left((2x)^2 - (y^4)^2\right) = 4(2x - y^4)(2x + y^4)$

2.

$$\left(\frac{36a^{-4}b^{10}c^2}{a^2c^{-6}}\right)^{-1/2} = \left(36a^{-6}b^{10}c^8\right)^{-1/2} = (36)^{-1/2}\left(a^{-6}\right)^{-1/2}\left(b^{10}\right)^{-1/2}\left(c^8\right)^{-1/2} =$$

$$= \frac{1}{(36)^{1/2}}a^3b^{-5}c^{-4} = \frac{1}{\sqrt{36}}\ \frac{a^3}{b^5c^4} = \frac{a^3}{6b^5c^4}$$

3.

$$\frac{3x}{3x+1} - \frac{x}{x-2} = \frac{3x}{(3x+1)}\ \frac{(x-2)}{(x-2)} - \frac{x}{(x-2)}\ \frac{(3x+1)}{(3x+1)} =$$

$$= \frac{3x(x-2) - x(3x+1)}{(3x+1)(x-2)} = \frac{3x^2 - 6x - 3x^2 - x}{(3x+1)(x-2)} = \frac{-7x}{(3x+1)(x-2)}$$

4.

$$(2x+1)^3(2)(3x-5)(3) + (3x-5)^2(3)(2x+1)^2(2)$$
$$= 6\left[(2x+1)^3(3x-5) + (3x-5)^2(2x+1)^2\right]$$
$$= 6(2x+1)^2\left[(2x+1)(3x-5) + (3x-5)^2\right]$$
$$= 6(2x+1)^2(3x-5)\left[(2x+1) + (3x-5)\right]$$
$$= 6(2x+1)^2(3x-5)(5x-4)$$
$$= 6(3x-5)(5x-4)(2x+1)^2$$

5. $$\frac{xy^{-1}}{(x+y)^{-1}} = \frac{\frac{x}{y}}{\frac{1}{(x+y)}} = \frac{x}{y}\ \frac{(x+y)}{1} = \frac{x(x+y)}{y}$$

6.

$$A = P(1 + rt)$$
$$A = P + \Pr t$$
$$A - P = \Pr t$$
$$\frac{A-P}{\Pr} = \frac{\Pr t}{\Pr}$$
$$\frac{A-P}{\Pr} = t$$
$$t = \frac{A-P}{\Pr}$$

7.

$$\frac{4}{2p-3} + \frac{10}{4p^2-9} = \frac{1}{2p+3}$$
$$\frac{4}{2p-3} + \frac{10}{(2p-3)(2p+3)} = \frac{1}{2p+3}$$
$$(2p-3)(2p+3)\left[\frac{4}{2p-3} + \frac{10}{(2p-3)(2p+3)}\right] = \left[\frac{1}{2p+3}\right](2p-3)(2p+3)$$
$$(2p+3)\ 4 + 10 = 1\ (2p-3)$$
$$4(2p+3) + 10 = (2p-3)$$
$$8p + 12 + 10 = 2p - 3$$
$$8p - 2p = -3 - 12 - 10$$
$$6p = -25$$
$$p = -\frac{25}{6}$$

8. $$\frac{\sqrt{x}+5}{\sqrt{x}-5} = \frac{(\sqrt{x}+5)}{(\sqrt{x}-5)}\frac{(\sqrt{x}+5)}{(\sqrt{x}+5)} =$$

$$= \frac{x + 5\sqrt{x} + 5\sqrt{x} + 25}{x + 5\sqrt{x} - 5\sqrt{x} - 25} = \frac{x + 10\sqrt{x} + 25}{x - 25}$$

9. t = # of hours the other person takes to complete the job.

fraction from 1st person + fraction from 2nd person=whole job

$$\left(\frac{1}{6}\ \frac{job}{hour}\right)4hours + \left(\frac{1}{t}\ \frac{job}{hour}\right)4hours = \left(\frac{1}{4}\ \frac{job}{hour}\right)4hours$$

$$\left(\frac{2}{3}\right)job + \left(\frac{4}{t}\right)job = 1job$$

$$\frac{2}{3} + \frac{4}{t} = 1$$

$$3t\left(\frac{2}{3} + \frac{4}{t}\right) = 1\ 3t$$

$$2t + 12 = 3t$$
$$12 = t$$
$$t = 12$$

10.

$$\begin{cases} y = x + 1 \\ y^2 - x^2 = 145 \end{cases}$$
$$(x+1)^2 - x^2 = 145$$
$$x^2 + 2x + 1 - x^2 = 145$$
$$2x + 1 = 145$$
$$2x = 144$$
$$x = 72$$

11. let t = # hours truck has been traveling

$$40t = 55(t-1)$$
$$40t = 55t - 55$$
$$55 = 15t$$

	rate	time	distance
truck	40	t	40t
car	55	t - 1	55(t – 1)

$t = \frac{55}{15} = \frac{11}{3}$ hours, so distance is $40\left(\frac{11}{3}\right) = \frac{440}{3}$ miles

12.

let $x =$ # ml of the 50% solution

let $y =$ total # of ml

$$\begin{cases} x + 40 = y \\ x(.50) + 40(.20) = y(.25) \end{cases}$$

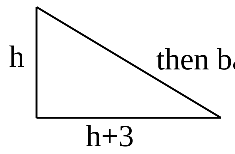
B

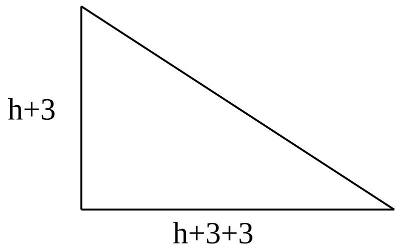
$$x(.50) + 8 = (x + 40)(.25)$$
$$.50x + 8 = .25x + 10$$
$$.25x = 2$$
$$x = 8 \text{ ml}$$

13.

Let h = height of original triangle

then base of original triangle = h+3





New:

Area of new triangle $= 14\ in^2$

$\frac{1}{2}(h+3)(h+6) = 14$

$(h+3)(h+6) = 28$

$h^2 + 3h + 6h + 18 = 28$

A

$h^2 + 9h - 10 = 0$

$(h+10)(h-1) = 0$

$h = -10, \quad h = 1$

Original height $= 1\ in.$

Original base $= 1 + 3 = 4\ in.$

14.

let t = number of years after 1980 and let V =value
t is the independent variable and V is the dependent variable

points on line $(1,54)$ and $(3,62)$

slope $m = \frac{62-54}{3-1} = \frac{8}{2} = 4$

$V - V_1 = m(t - t_1)$

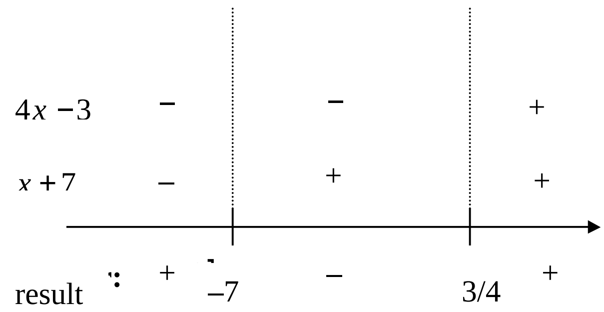
$V - 54 = 4(t-1)$

$V - 54 = 4t - 4$

$V = 4t + 50$

A

15. $(4x-3)(x+7) \le 0$



16.

$|6 - 2x| \le 3$

$-3 \le 6 - 2x \le 3$

$-9 \le -2x \le -3$

$\frac{9}{2} \ge x \ge \frac{3}{2}$

C

$\frac{3}{2} \le x \le \frac{9}{2}$

17.

$A\ (1,-2)$, Midpoint $M\ (2,3)$, $B\ (x,y)$

Midpoint $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Midpoint of $\overline{AB}$ $\left(\frac{1+x}{2}, \frac{-2+y}{2}\right)$ $(2,3)$

$\frac{1+x}{2} = 2, \qquad \frac{-2+y}{2} = 3$

$1 + x = 4, \qquad -2 + y = 6$

$x = 3, \qquad y = 8$

so $B\ (3,8)$

C

18.

slope of line $m = -\frac{1}{3}$

slope of line perpendicular $m = 3$

D

19.

$2x - 3y = 7$

$-3y = -2x + 7$

$y = \frac{2}{3}x - \frac{7}{3}$

slope $m = \frac{2}{3}$

slope of parallel line $m = \frac{2}{3}$

point is $(2,-1)$; $m = \frac{2}{3}$

$y = mx + b$

$-1 = \left(\frac{2}{3}\right)(2) + b$ **C**

$-1 = \frac{4}{3} + b$

$b = -\frac{7}{3}$ so $y = \frac{2}{3}x - \frac{7}{3}$

20.

Center $(0,2)$ $(x-h)^2 + (y-k)^2 = r^2$

radius = 2 $(x-0)^2 + (y-2)^2 = 2^2$

$x^2 + (y-2)^2 = 4$ **B**

$x^2 + y^2 - 4y + 4 = 4$

$x^2 + y^2 - 4y = 0$

21.

$f(x) = 1 - \sqrt{x}$, $g(x) = \frac{1}{x}$

D

$(g \circ f)(x) = g[f(x)] = g(1-\sqrt{x}) = \frac{1}{1-\sqrt{x}}$

22.

$f(x) = \frac{x}{x^2+1}$

D

$\frac{1}{f(3)} = \frac{1}{\frac{3}{(3)^2+1}} = \frac{1}{\frac{3}{10}} = \frac{10}{3}$

23.

$y = \frac{1}{3x-2}$

$x = \frac{1}{3y-2}$

$x(3y-2) = 1$

$3xy - 2x = 1$

$3xy = 1 + 2x$

$y = \frac{1+2x}{3x} = f^{-1}(x)$

24.

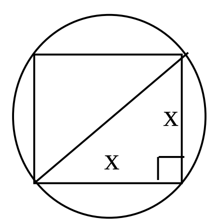
$f(x) = x^2 - 2x + 4$

$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 2(x+h) + 4 - (x^2 - 2x + 4)}{h}$ **A**

$= \frac{x^2 + 2xh + h^2 - 2x - 2h + 4 - x^2 + 2x - 4}{h} = \frac{2xh + h^2 - 2h}{h}$

$= \frac{h(2x + h - 2)}{h} = 2x + h - 2$

25.



Let A = area of circle

Area of circle $A(r) = \pi r^2$

Diameter (d) of circle $x^2 + x^2 = d^2$

$2x^2 = d^2$ **A**

$d = \pm\sqrt{2x^2}$

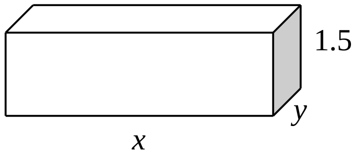
$d = x\sqrt{2}$

Radius (r) of circle $r = \frac{x\sqrt{2}}{2}$

So, $A(x) = \pi\left(\frac{x\sqrt{2}}{2}\right)^2 = \pi\left(\frac{x^2(2)}{4}\right)$

$= \frac{\pi x^2}{2}$ or $\frac{\pi}{2}x^2$

26.



Volume $= 6\,ft.^3$

$xy(1.5) = 6$

$y = \dfrac{6}{1.5x}$ **B**

$y = \dfrac{4}{x}$

27.

$T = k\dfrac{a^3}{\sqrt{d}}$

$4 = k\dfrac{2^3}{\sqrt{9}}$

$4 = k\dfrac{8}{3}$

$k = \dfrac{4}{1}\ \dfrac{3}{8}$

$k = \dfrac{3}{2}$

$T = \dfrac{3}{2}\ \dfrac{(-1)^3}{\sqrt{4}}$

$T = \dfrac{3}{2}\left(-\dfrac{1}{2}\right)$ **A**

$T = -\dfrac{3}{4}$

28.

$x^2 - 4x - 2y - 4 = 0$

$2y = x^2 - 4x - 4$

$2y = (x^2 - 4x + 4) - 4 - 4$

$2y = (x-2)^2 - 8$

$y = \frac{1}{2}(x-2)^2 - 4$

$y = a(x-h)^2 + k$

$Vertex(h,k) = (2,-4)$

29.

Vertex $V(0,2)$

point on parabola $(1,0)$

$y = a(x-h)^2 + k$

$y = a(x-0)^2 + 2$

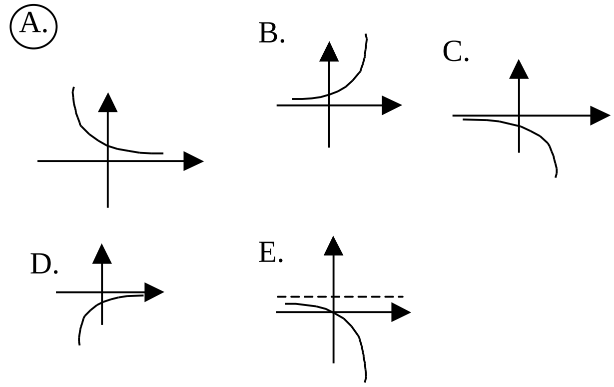
$y = ax^2 + 2$ **B**

$0 = a(1)^2 + 2$

$a = -2$

$y = -2x^2 + 2$

30.



31.

$\log_b y^3 + \log_b y^2 - \log_b y^4 = \log_b(y^3y^2) - \log_b y^4$

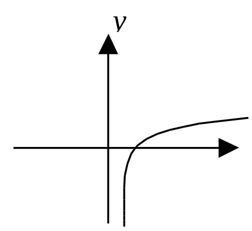
$= \log_b y^5 - \log_b y^4 = \log_b\left(\dfrac{y^5}{y^4}\right) = \log_b y$ **B**

32.

$f(x) = \log_a x$ if $a > 1$

example: if $a = 2$, then $f(x) = \log_2 x$, **D**

Graph of $y = \log_2 x$ $\quad 2^y = x$



f is increasing, f does not have a as an x-intercept (the x-int. is (1,0)), f does not have a y-intercept, the domain of f is $(0,\infty)$.

33.

$\log\left(\dfrac{432}{\left(\sqrt{.095}\right)\left(\sqrt[3]{72.1}\right)}\right) = \log\left(\dfrac{432}{(.095)^{\frac{1}{2}}(72.1)^{\frac{1}{3}}}\right)$

$= \log 432 - \left(\log\left[(.095)^{\frac{1}{2}}(72.1)^{\frac{1}{3}}\right]\right)$ **B**

$= \log 432 - \left(\log(.095)^{\frac{1}{2}} + \log(72.1)^{\frac{1}{3}}\right)$

$= \log 432 - \dfrac{1}{2}\log .095 - \dfrac{1}{3}\log 72.1$

34.

$\log_x 2 = 5$

$x^5 = 2$

$\left(x^5\right)^{\frac{1}{5}} = (2)^{\frac{1}{5}}$ **D**

$x = \sqrt[5]{2}$

$x \approx 1.1487$

35.

$\frac{\log_5\left(\frac{1}{8}\right)}{\log_5(2)} = \log_2\left(\frac{1}{8}\right) = \log_2\left(2^{-3}\right) = -3$

36.

$3^{x-5} = 4$

$\log 3^{-5} = \log 4$

$(x-5)\log 3 = \log 4$ **C**

$x - 5 = \frac{\log 4}{\log 3}$

$x = \frac{\log 4}{\log 3} + 5$

37.

$\log_3 \sqrt{2x+3} = 2$

$3^2 = \sqrt{2x+3}$

$\sqrt{2x+3} = 9$ **C**

$\left(\sqrt{2x+3}\right)^2 = (9)^2$ Check : $\sqrt{2(39)+3} = 9$

$2x + 3 = 81$ $9 = 9$

2x = 78 Check: $\log_3 \sqrt{2(39)+3} = 2$

$x = 39$ $3^2 = \sqrt{81}$

38.

$\log_3 m = 8$

$\log_3 n = 10$

$\log_3 p = 6$

$\log_3\left(\frac{\sqrt{mn}}{p^3}\right) = \log_3(mn)^{\frac{1}{2}} - \log_3 p^3$

$= \log_3\left(m^{\frac{1}{2}} n^{\frac{1}{2}}\right) - \log_3 p^3$ **A**

$= \log_3 m^{\frac{1}{2}} + \log_3 n^{\frac{1}{2}} - \log_3 p^3$

$= \frac{1}{2}\log_3 m + \frac{1}{2}\log_3 n - 3\log_3 p$

$= \frac{1}{2}(8) + \frac{1}{2}(10) - 3(6)$

$= 4 + 5 - 18 = -9$

39. Half-life means when half of te initial amount still remains, $\frac{1}{2}q_o$.

$\frac{1}{2}q_o = q_o e^{-0.0063t}$

$\frac{1}{2} = e^{-0.0063t}$

$\ln\left(\frac{1}{2}\right) = \ln\left(e^{-0.0063t}\right)$

$\ln\left(\frac{1}{2}\right) = -0.0063t$

$\frac{\ln(0.5)}{-0.0063} = t \approx 110.0$ days

40.

$y = 2 + 2^x$

When $x = 0$, $y = 2 + 2^0$ **D**

$y = 2 + 1 = 3$

41.

$\begin{cases} x + 4y = 3 \\ 2x - 6y = 8 \end{cases}$ $x = 3 - 4y$

$2(3-4y) - 6y = 8$

$6 - 8y - 6y = 8$

$y = \frac{2}{-14} = -\frac{1}{7}$ $x = 3 - 4\left(-\frac{1}{7}\right) = \frac{25}{7}$

$\left(\frac{25}{7}, -\frac{1}{7}\right)$

42.

$$\begin{cases} x^2 + y^2 = 16 \\ 2y - x = 4 \end{cases} \qquad x = 2y - 4$$

$$(2y-4)^2 + y^2 = 16$$

$$4y^2 - 16y + 16 + y^2 = 16$$

$$5y^2 - 16y = 0$$

$$y(5y - 16) = 0$$

$$y = 0 \qquad x = 2(0) - 4 = -4$$

$$y = \tfrac{16}{5} \qquad x = 2\left(\tfrac{16}{5}\right) - 4 = \tfrac{12}{5}$$

$$(-4,0)\ \&\ \left(\tfrac{12}{5}, \tfrac{16}{5}\right)$$

43.

$$\begin{cases} x + y - z = -1 \\ 4x - 3y + 2z = 16 \\ 2x - 2y - 3z = 5 \end{cases} \qquad x = -y + z - 1$$

$$\begin{cases} 4(-y + z - 1) - 3y + 2z = 16 \\ 2(-y + z - 1) - 2y - 3z = 5 \end{cases}$$

$$\begin{cases} -7y + 6z - 4 = 16 \\ -4y - z - 2 = 5 \end{cases}$$

$$\begin{cases} -7y + 6z = 20 \\ -4y - z = 7 \end{cases} \qquad z = -4y - 7$$

$$-7y + 6(-4y - 7) = 20$$

$$-31y = 62$$

$$y = -2$$

$$y = -2 \qquad z = -4(-2) - 7$$

$$z = 1$$

44.

$$x^2 + 6x + 34$$

$$x^2 - 6x + 0 \,\overline{)\, x^4 + 0x^3 - 2x^2 + 0x - 3}$$

$$+\left(^{-}x^4 + 6x^3 + {}^{-}0x^2\right)$$

$$6x^3 - 2x^2 + 0x$$

$$+\left(^{-}6x^3 + 36x^2 + {}^{-}0x\right)$$

$$34x^2 + 0x - 3$$

$$+\left(^{-}34x^2 + 204x + {}^{-}0\right)$$

$$204x - 3$$

C

$$q(x) = x^2 + 6x + 34$$

$$r(x) = 204x - 3$$

45. If the denominator of the function is equal to zero the function will be undefined.

$$f(x) = \frac{(x+3)(x-3)}{x(x+2)}$$

when $x = 0$ or $x = -2$

46.

$$y = x^2(x-1)(x+1)^2$$

x-intercepts: $x^2(x-1)(x+1)^2 = 0$

$$x = 0,\ x = 1,\ x = -1$$

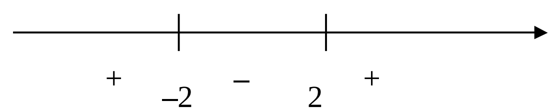
A

Interval	$(-\infty, -1)$	$(-1,0)$	$(0,1)$	$(1,\infty)$
x^2	+	+	+	+
$x-1$	−	−	−	+
$(x+1)^2$	+	+	+	+
Result	− below x-axi	− below x-axi	− below x-axi	+ above x-axi

47.

$x = 2$ x-intercept

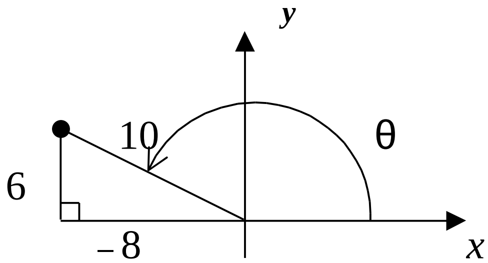
$x = -2$ vertical asymptote



E is the closest answer. The scale is a bit off on the x-axis.

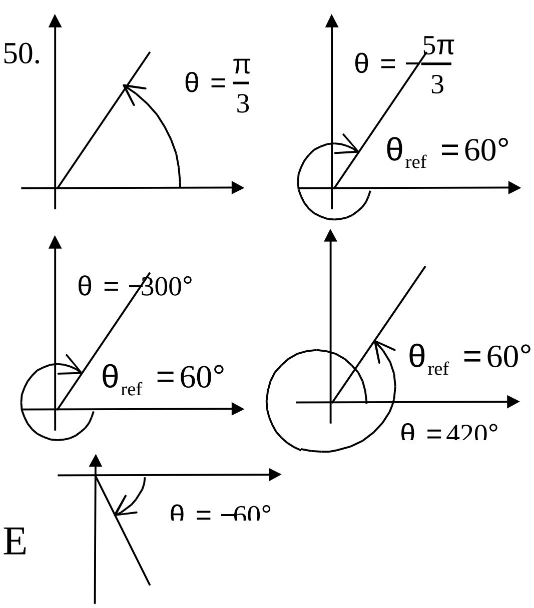
48. Shifted left 1 unit, then reflected about x-axis, then shifted down 2 units -- Answer: C

49.



$$\sin\theta = \frac{6}{10} = \frac{y}{r}$$
$$x^2 + y^2 = r^2$$
$$x^2 + 6^2 = 10^2$$
$$x^2 = 64$$
$$x = \pm 8$$
$$x = -8$$
$$\cos\theta = \frac{x}{r} = \frac{-8}{10} = -0.8$$

50.



E

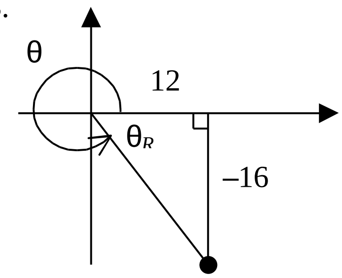
51.

$$135° \; \frac{\pi \text{ radians}}{180°} = \frac{3\pi}{4}$$

52.

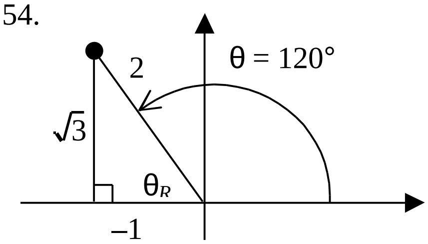
$$\sec 126° = \frac{1}{\cos 126°} \approx \frac{1}{-0.587785} \approx -1.7013$$

53.



$$\tan\theta = \frac{y}{x} = \frac{-16}{12} = -\frac{4}{3}$$

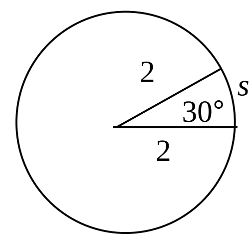
54.



$\theta_R = 60°$

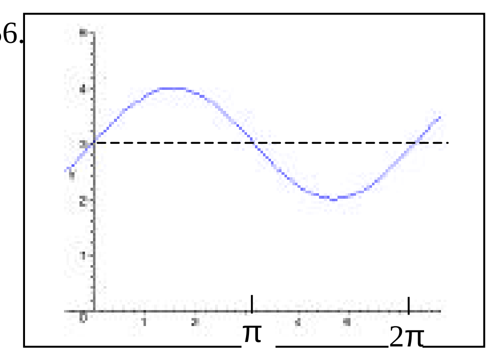
$$\tan\theta = \frac{y}{x} = \frac{\sqrt{3}}{-1} = -\sqrt{3}$$

55.



$$s = r\theta = 2\left(\frac{\pi}{6}\right) = \frac{\pi}{3} \approx 1.047$$

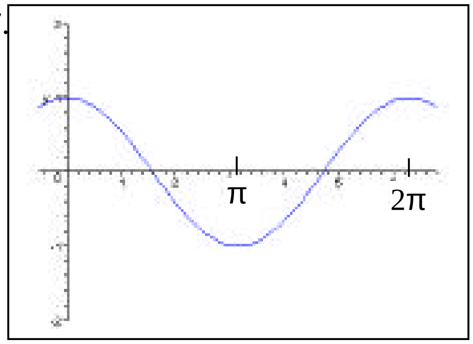
56.



The graph is the $y = \sin x$ shifted up three units.

I(yes), II(no), III(yes), IV(yes), **B**

57.



D = all real number = $(-\infty, \infty)$

R = all possible outputs/y-values = $[-1, 1]$

58.

$$\frac{\tan^2 x}{1+\sec x} = \frac{\sec^2 x - 1}{\sec x + 1} = \frac{(\sec x + 1)(\sec x - 1)}{(\sec x + 1)}$$

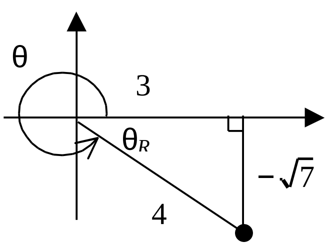
$= \sec x - 1$, remember $\tan^2 x = (\tan x)^2$

59.

$$\frac{\tan x \;\; \cos x \;\; \csc x}{\cot x \;\; \sec x \;\; \sin x} = \frac{\tan x \;\; \tan x \;\; \cos x \;\; \cos x}{\sin x \;\; \sin x}$$

$$= \frac{\tan^2 x \;\; \cos^2 x}{\sin^2 x} = \frac{\sin^2 x}{\cos^2 x} \; \frac{\cos^2 x}{\sin^2 x} = 1$$

60.



$x^2 + y^2 = r^2$

$y^2 = 4^2 - 3^2$

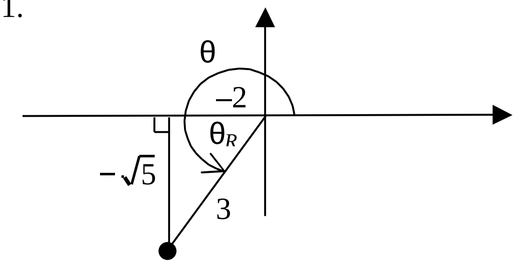
$y = \pm\sqrt{7}$

$y = -\sqrt{7}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{-\sqrt{7}}{4}\right)\left(\frac{3}{4}\right) = -\frac{3\sqrt{7}}{8}$

61.



Given : $\tan\theta = \frac{\sqrt{5}}{2} = \frac{-\sqrt{5}}{-2} = \frac{y}{x}$

$\frac{\theta}{2}$ is in QII, $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$

$\cos\left(\frac{\theta}{2}\right) = -\sqrt{\frac{1+\left(\frac{-2}{3}\right)}{2}} = -\sqrt{\frac{1-\frac{2}{3}}{2}}$

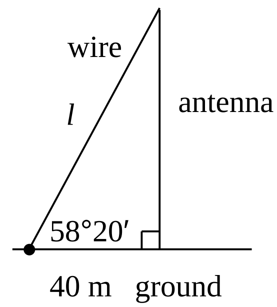
62.

$\cos 58.\overline{3}^\circ = \frac{40}{l}$

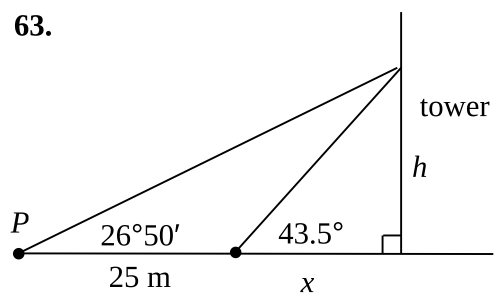
$l \;\; \cos 58.\overline{3}^\circ = 40$

$l = \frac{40}{\cos 58.\overline{3}^\circ}$

$l \approx 76.2$ m



63.



$\tan 43.5^\circ = \frac{h}{x}$, $\quad \tan 26.8\overline{3}^\circ = \frac{h}{x+25}$

$h = x \;\; \tan 43.5^\circ$

$\tan 26.8\overline{3}^\circ = \frac{x \;\; \tan 43.5^\circ}{x+25}$

$\tan 26.8\overline{3}^\circ (x+25) = x \;\; \tan 43.5^\circ$

$x \tan 26.8\overline{3}^\circ + 25\tan 26.8\overline{3}^\circ = x \;\; \tan 43.5^\circ$

$25\tan 26.8\overline{3}^\circ = x \;\; \tan 43.5^\circ - x\tan 26.8\overline{3}^\circ$

$x = \frac{25\tan 26.8\overline{3}^\circ}{\tan 43.5^\circ - \tan 26.8\overline{3}^\circ} \approx 28.541487$

$h = x \;\; \tan 43.5^\circ \approx 27.1$ meters

64. Examine r when $\theta = 0$ and as $\theta \to 90°$

A. $r = 1$ when $\theta = 0$ and as $\theta \to 90°$ $r \to 2$, looks right as the angle changes from 0° to 90°.

B. $r = 2$ when $\theta = 0$ and as $\theta \to 90°$ $r \to 1$, looks incorrect as the angle changes from 0° to 90°. The radial distance should be getting bigger.

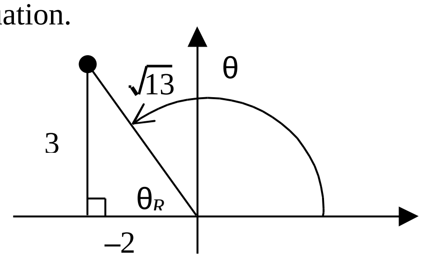
C. $r = 1$ when $\theta = 0$ and as $\theta \to 90°$ $r \to 0$, looks incorrect as the angle changes from 0° to 90°. The radial distance should be getting bigger.

D. $r = 2$ when $\theta = 0$ and as $\theta \to 90°$ $r \to 0$, looks incorrect as the angle changes from 0° to 90°. The radial distance should be getting bigger.

E. $r = 0$ when $\theta = 0$ and as $\theta \to 90°$ $r \to 2$, looks incorrect as the angle changes from 0° to 90°. When the angle is zero the radial distance should greater than zero.

Plugging in further angles would yield more points that will confirm that **A** is the correct polar equation.

65.



$$r^2 = \left(-2^2\right) + 3^2$$

$$r = \sqrt{13}$$

$$\tan\theta = \frac{y}{x} = \frac{3}{-2} \qquad \tan^{-1}\left(-\frac{3}{2}\right) \approx -56.301°$$

$$\theta_R = +56.301°$$

$$\theta = 180° - \theta_R \approx 123.7°$$

$$\left(\sqrt{13}, 123.7°\right)$$

66.

$$x^2 - 2x + y^2 = 0$$

$$x^2 + y^2 - 2x = 0$$

$$r^2 - 2r\cos\theta = 0$$

$$r\left(r - 2\cos\theta\right) = 0$$

$r = 0$, which is not an equation of the given circle

or $\underline{r = 2\cos\theta}$