Solutions to the Practice Questions for the Final Exam
MA 153
Fall 2001

1.

$$\frac{\frac{15}{5}}{\frac{1}{2}} = \frac{3}{\frac{1}{2}} = \frac{3}{1}\ \frac{2}{1} = 6$$ **D**

2.

$$16x^2 - 4y^8 = 4(4x^2 - y^8) = 4\left[(2x)^2 - \left(y^4\right)^2\right]$$ **C**

$$= 4\left(2x + y^4\right)\left(2x - y^4\right)$$

3.

$$\left(\frac{4a^4b^8}{c^{-2}}\right)^{-\frac{1}{2}} = \left(4a^4b^8c^2\right)^{-\frac{1}{2}} = \left(\frac{1}{4a^4b^8c^2}\right)^{\frac{1}{2}}$$ **A**

$$= \frac{\sqrt{1}}{\sqrt{4a^4b^8c^2}} = \frac{1}{2a^2b^4c}$$

4.

$$\frac{3x}{3x+1} - \frac{x}{x-2} = \frac{3x(x-2)}{(3x+1)(x-2)} - \frac{x(3x+1)}{(3x+1)(x-2)}$$ **C**

$$= \frac{\cancel{3x^2} - 6x - \cancel{3x^2} - x}{(3x+1)(x-2)} = \frac{-7x}{(3x+1)(x-2)}$$

5.

$$\frac{x-2}{(x+1)(x-3)} \div \frac{(x-2)(x+1)}{(x+3)(x-3)}$$ **B**

$$= \frac{\cancel{x-2}}{(x+1)\cancel{(x-3)}}\ \frac{(x+3)\cancel{(x-3)}}{\cancel{(x-2)}(x+1)} = \frac{x+3}{(x+1)^2}$$

6.

let $t =$ time second person working alone

$\frac{1}{6}+\frac{1}{t}=\frac{1}{4}$ or $\frac{1}{6}(4)+\frac{1}{t}(4)=1$

$12t\left[\frac{1}{6}+\frac{1}{t}\right]=\left[\frac{1}{4}\right]12t$ $\quad 3t\left[\frac{2}{3}+\frac{4}{t}\right]=[1]3t$ **E**

$2t+12=3t$ $\quad 2t+12=3t$

$t=12$ $\quad t=12$

7. $\frac{xy^{-1}}{(x+y)^{-1}}=\frac{\frac{x}{y}}{\frac{1}{(x+y)}}=\frac{x}{y}\ \frac{(x+y)}{1}=\frac{x(x+y)}{y}$ **A**

8.

$$\frac{\sqrt{3}}{2+\sqrt{3}}\ \frac{(2-\sqrt{3})}{(2-\sqrt{3})}=\frac{\sqrt{3}\left(2-\sqrt{3}\right)}{\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)}$$

C

$$=\frac{2\sqrt{3}-\left(\sqrt{3}\right)^2}{(2)^2-\left(\sqrt{3}\right)^2}=\frac{2\sqrt{3}-3}{4-3}=2\sqrt{3}-3$$

9.

let $x =$ first positive integer $\left(x<y\right)$

$\begin{cases} y=x+1 \\ y^2-x^2=145 \end{cases}$ substitute eq. (1) into eq.(2) for y:

$\left(x+1\right)^2-x^2=145$

$x^2+2x+1-x^2=145$ **B**

$2x+1=145$

$2x=144$

$x=72$

10.

$A = P(1 + rt)$

$A = P + \text{Pr}\,t$

$A - P = \text{Pr}\,t$ **E**

$\frac{A - P}{\text{Pr}} = t$

11. let t = # hours truck has been traveling

$40t = 55(t - 1)$

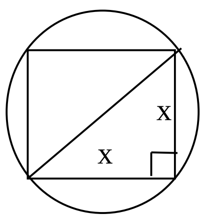
$40t = 55t - 55$

$55 = 15t$

	rate	time	distance
truck	40	t	40t
car	55	t - 1	55(t – 1)

$t = \frac{55}{15} = \frac{11}{3}$ hours, so distance is $40\left(\frac{11}{3}\right) = \frac{440}{3}$ miles **A**

12.



Let A = area of circle

Area of circle $A(r) = \pi r^2$

Diameter (d) of circle $x^2 + x^2 = d^2$

$2x^2 = d^2$

$d = \pm\sqrt{2x^2}$

$d = x\sqrt{2}$ **A**

Radius (r) of circle $r = \frac{x\sqrt{2}}{2}$

So, $A(x) = \pi\left(\frac{x\sqrt{2}}{2}\right)^2 = \pi\left(\frac{x^2(2)}{4}\right)$

$= \frac{\pi x^2}{2}$ or $\frac{\pi}{2}x^2$

13.

$$\frac{4}{2p-3}+\frac{10}{4p^2-9}=\frac{1}{2p+3} \qquad \left(Domain: p \neq \pm\frac{3}{2}\right)$$

$$(2p+3)(2p-3)\left[\frac{4}{2p-3}+\frac{10}{(2p+3)(2p-3)}\right]=\left[\frac{1}{2p+3}\right](2p+3)(2p-3)$$

$$4(2p+3)+10=2p-3$$

D

$$8p+12+10=2p-3$$

$$6p=-25$$

$$p=-\frac{25}{6}$$

14.

let $x =$ # ml of the 50% solution

let $y =$ total # of ml

$$\begin{cases} x+40=y \\ x(.50)+40(.20)=y(.25) \end{cases}$$

B

$x(.50)+8=(x+40)(.25)$

$.50x+8=.25x+10$

$.25x=2$

$x=8$ ml

15.

$x=\sqrt{14+5x}$ Check:

$(x)^2=\left(\sqrt{14+5x}\right)^2$ If $x=7$:

$x^2=14+5x$ $7=\sqrt{14+35}$

$x^2-5x-14=0$ $7=\sqrt{49}$ yes

E $[x=7]$

$(x-7)(x+2)=0$ If $x=-2$:

$x=7$, ~~$x=-2$~~ $-2=\sqrt{14-10}$

$-2=\sqrt{4}$ no

16

$m^4 - m^2 - 6 = 0$

$(m^2 - 3)(m^2 + 2) = 0$

$m^2 - 3 = 0, \qquad m^2 + 2 = 0$ **D**

$m^2 = 3, \qquad m^2 = -2$

$m = \pm\sqrt{3}, \qquad m = \pm\sqrt{-2}$

$m = \pm i\sqrt{2}$

17.

$3x - 2 > 6x + 1$

$-3x > 3$

$x < -1$ **A**

$(-\infty, \ -1)$

18.

$|6 - 2x| \le 3$

$-3 \le 6 - 2x \le 3$

$-9 \le -2x \le -3$

$\frac{9}{2} \ge x \ge \frac{3}{2}$ **C**

$\frac{3}{2} \le x \le \frac{9}{2}$

19.

$2x^2 - 4x + k = 0$

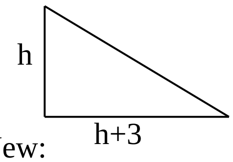
$x = \frac{4 \pm \sqrt{16 - 4(2)(k)}}{2(2)} = \frac{4 \pm \sqrt{16 - 8k}}{4}$

Need : $16 - 8k \ge 0$ **D**

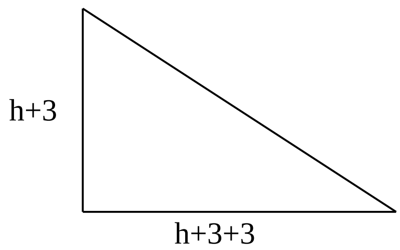
$-8k \ge -16$

$k \le 2$

20.



New:



Let h = height of original triangle
then base of original triange = h+3

Area of new triangle $= 14 \ in^2$

$$\frac{1}{2}(h+3)(h+6) = 14$$

$$(h+3)(h+6) = 28$$

$$h^2 + 3h + 6h + 18 = 28$$

$$h^2 + 9h - 10 = 0$$

A

$$(h+10)(h-1) = 0$$

$$\cancel{h = -10}, \quad h = 1$$

Original height $= 1 \ in.$

Original base $= 1 + 3 = 4 \ in.$

21.

$$\begin{cases} 2x^2 + y^2 = 1 \\ x - y = 1 \end{cases} \qquad y = x - 1$$

$$2x^2 + (x-1)^2 = 1$$

$$2x^2 + x^2 - 2x + 1 = 1$$

B

$$3x^2 - 2x = 0$$

$$x(3x-2) = 0$$

$$x = 0, \ x = \frac{2}{3}$$

22.

$A(1,-2)$, Midpoint $M(2,3)$, $B(x,y)$

Midpoint $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Midpoint of $\overline{AB}$ $\left(\frac{1+x}{2}, \frac{-2+y}{2}\right)$ $(2,3)$

$\frac{1+x}{2}=2,\quad \frac{-2+y}{2}=3$ **C**

$1+x=4,\quad -2+y=6$

$x=3,\quad y=8$

so $B(3,8)$

23.

slope of line $m=-\frac{1}{3}$ **D**

slope of line perpendicular $m=3$

24.

$m=\frac{xyk}{z}$ $3=\frac{(4)(2)k}{6}$

$18=8k$ **B**

$k=\frac{18}{8}=\frac{9}{4}$

25.

$2x-3y=7$

$-3y=-2x+7$

$y=\frac{2}{3}x-\frac{7}{3}$

slope $m=\frac{2}{3}$

slope of parallel line $m=\frac{2}{3}$

point is $(2,-1)$; $m=\frac{2}{3}$

$y=mx+b$

$-1=\left(\frac{2}{3}\right)(2)+b$ **C**

$-1=\frac{4}{3}+b$

$b=-\frac{7}{3}$ so $y=\frac{2}{3}x-\frac{7}{3}$

26.

Center $(0,2)$ $(x-h)^2+(y-k)^2=r^2$

radius = 2 $(x-0)^2+(y-2)^2=2^2$

$x^2+(y-2)^2=4$ **B**

$x^2+y^2-4y+4=4$

$x^2+y^2-4y=0$

27.

$f(x)=1-\sqrt{x},\qquad g(x)=\frac{1}{x}$

D

$(g\circ f)(x)=g[f(x)]=g(1-\sqrt{x})=\frac{1}{1-\sqrt{x}}$

28.

$f(x)=\frac{x}{x^2+1}$

$\frac{1}{f(3)}=\frac{1}{\frac{3}{(3)^2+1}}=\frac{1}{\frac{3}{10}}=\frac{10}{3}$ **D**

29.

Vertex $V(0,2)$ $y=a(x-h)^2+k$

point on parabola $(1,0)$ $y=a(x-0)^2+2$

$y=ax^2+2$

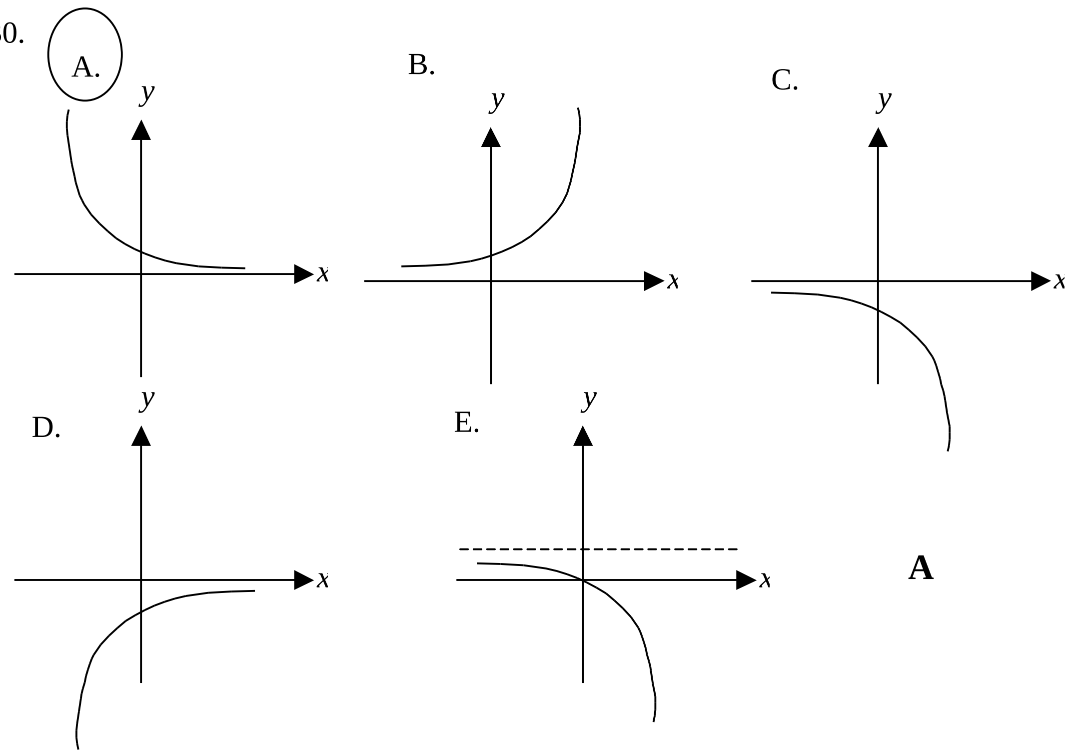
B

$0=a(1)^2+2$

$a=-2$

$y=-2x^2+2$

30.



A

31.

$$\log_b y^3 + \log_b y^2 - \log_b y^4 = \log_b(y^3 y^2) - \log_b y^4$$

$$= \log_b y^5 - \log_b y^4 = \log_b\left(\frac{y^5}{y^4}\right) = \log_b y$$

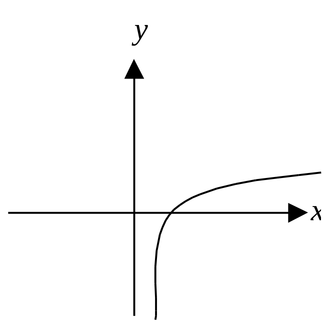
B

32.

$f(x) = \log_a x$ if $a > 1$

example: if $a = 2$, then $f(x) = \log_2 x$,

Graph of $y = \log_2 x$ $\quad 2^y = x$



f is increasing, f does not have a as an x-intercept (the x-int. is (1,0)), f does not have a y-intercept, the domain of f is $(0, \infty)$.

D

33.

$$\log\left(\frac{432}{\left(\sqrt{.095}\right)\left(\sqrt[3]{72.1}\right)}\right)=\log\left(\frac{432}{(.095)^{\frac{1}{2}}(72.1)^{\frac{1}{3}}}\right)$$

$$=\log 432-\left(\log\left[(.095)^{\frac{1}{2}}(72.1)^{\frac{1}{3}}\right]\right)$$

B

$$=\log 432-\left(\log(.095)^{\frac{1}{2}}+\log(72.1)^{\frac{1}{3}}\right)$$

$$=\log 432-\frac{1}{2}\log .095-\frac{1}{3}\log 72.1$$

34.

$$3^{x-5}=4$$

$$\log 3^{x-5}=\log 4$$

$$(x-5)\log 3=\log 4$$

C

$$x-5=\frac{\log 4}{\log 3}$$

$$x=\frac{\log 4}{\log 3}+5$$

35.

$$\log_3\sqrt{2x+3}=2$$

$$3^2=\sqrt{2x+3}$$

$$\sqrt{2x+3}=9$$

$$\left(\sqrt{2x+3}\right)^2=(9)^2$$

Check : $\sqrt{2(39)+3}=9$

C

$$2x+3=81$$

$9=9$

$$2x=78$$

Check: $\log_3\sqrt{2(39)+3}=2$

$$x=39$$

$3^2=\sqrt{81}$

36.

$\log_3 m = 8$

$\log_3 n = 10$

$\log_3 p = 6$

$$\log_3\left(\frac{\sqrt{mn}}{p^3}\right) = \log_3(mn)^{\frac{1}{2}} - \log_3 p^3$$

$$= \log_3\left(m^{\frac{1}{2}} n^{\frac{1}{2}}\right) - \log_3 p^3$$

$$= \log_3 m^{\frac{1}{2}} + \log_3 n^{\frac{1}{2}} - \log_3 p^3$$

A

$$= \frac{1}{2}\log_3 m + \frac{1}{2}\log_3 n - 3\log_3 p$$

$$= \frac{1}{2}(8) + \frac{1}{2}(10) - 3(6)$$

$$= 4 + 5 - 18 = -9$$

37.

$y = 2 + 2^x$

When $x = 0$, $y = 2 + 2^0$

D

$y = 2 + 1 = 3$

38.

$y = x^2(x-1)(x+1)^2$

x-intercepts: $x^2(x-1)(x+1)^2 = 0$

$x = 0,\ x = 1,\ x = -1$

A

Interval	$(-\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
x^2	+	+	+	+
$x-1$	−	−	−	+
$(x+1)^2$	+	+	+	+
Result	− below x-axi	− below x-axi	− below x-axi	+ above x-axi

39.

$$\begin{cases} 2x-3y=4 \\ -4x+6y=3 \end{cases} \quad 2E_1 \begin{cases} 4x-6y=8 \\ -4x+6y=3 \end{cases} \quad E_1+E_2$$

Answer is C because:

$$\begin{cases} 4x-6y=8 \\ 0=11 \end{cases} \quad \text{No Solution}$$

40.

$$\begin{cases} x+4y=3 \\ 2x-6y=8 \end{cases} \quad -2E_1 \begin{cases} -2x-8y=-6 \\ 2x-6y=8 \end{cases} \quad E_1+E_2$$

$$\begin{cases} -2x-8y=-6 \\ -14y=2 \end{cases} \quad y=-\frac{2}{14}=-\frac{1}{7}$$

E

$$x+4\left(-\frac{1}{7}\right)=3$$

$$x=\frac{25}{7} \quad \text{so } \left(\frac{25}{7},-\frac{1}{7}\right)$$

41.

let t = number of years after 1980 and let V =value

t is the independent variable and V is the dependent variable

points on line $(1,54)$ and $(3,62)$

slope $m=\frac{62-54}{3-1}=\frac{8}{2}=4$

$V-V_1=m(t-t_1)$

A

$V-54=4(t-1)$

$V-54=4t-4$

$V=4t+50$

42.

$f(x) = x^2 - 2x + 4$

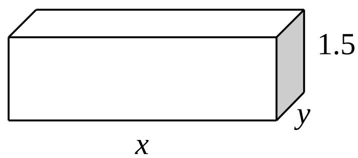
$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 2(x+h) + 4 - (x^2 - 2x + 4)}{h}$$

A

$$= \frac{x^2 + 2xh + h^2 - 2x - 2h + 4 - x^2 + 2x - 4}{h} = \frac{2xh + h^2 - 2h}{h}$$

$$= \frac{h(2x + h - 2)}{h} = 2x + h - 2$$

43.



Volume $= 6\,ft.^3$

$xy(1.5) = 6$

B

$y = \dfrac{6}{1.5x}$

$y = \dfrac{4}{x}$

44.

$\log_x 2 = 5$

$x^5 = 2$

$\left(x^5\right)^{\frac{1}{5}} = (2)^{\frac{1}{5}}$

D

$x = \sqrt[5]{2}$

$x \approx 1.1487$