MA 154 Exam 2 Key Fall 2001

Place your answers in the spaces provided. You must show correct work to receive credit.

(8 pts.) 1. Find all the solutions of the equation that are in the interval $[0°, 360°)$. Round the answer(s) to the nearest $0.01°$.

$$\csc\theta = -2.15$$

$\sin\theta = -0.4651$

$\theta_R = 27.72°$

Sine is negative in QIII and QIV

$\theta = 180° + 27.72° = 207.72°$

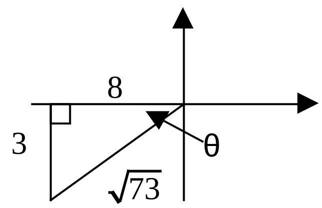
$\theta = 360° - 27.72° = 332.28°$

$207.72°,\ 332.28°$

(8 pts.) 2. Find the exact value of $\sin(2\theta)$ if $\cot\theta = \frac{8}{3}$ and $180° < \theta < 270°$.

$c^2 = 3^2 + 8^2$

$c = \sqrt{73}$



$\sin(2\theta) = 2\sin\theta\cos\theta$

$\sin(2\theta) = 2\left(-\frac{3}{\sqrt{73}}\right)\left(-\frac{8}{\sqrt{73}}\right)$

$\sin(2\theta) = \frac{48}{73}$

$\frac{48}{73}$

(10 pts.) 3. Find all the solutions of the equation that are in the interval $[0, 2\pi)$.

$$1 + \sin\theta = 2\left(1 - \sin^2\theta\right)$$

$1 + \sin\theta = 2 - 2\sin^2\theta$

$2\sin^2\theta + \sin\theta - 1 = 0$

$(2\sin\theta - 1)(\sin\theta + 1) = 0$

$2\sin\theta - 1 = 0$ $\quad$ $\sin\theta + 1 = 0$

$2\sin\theta = 1$ $\quad$ $\sin\theta = -1$

$\sin\theta = \frac{1}{2}$ $\quad$ $\theta = \frac{3\pi}{2}$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

$1 + \sin\theta = 2 - 2\sin^2\theta$

$2\sin^2\theta + \sin\theta - 1 = 0$

$\sin\theta = \frac{-1 \pm \sqrt{1 - 4(2)(-1)}}{2(2)}$

$\sin\theta = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1 \pm \sqrt{9}}{4} = \frac{-1 \pm 3}{4}$

$\sin\theta = \frac{2}{4} = \frac{1}{2}$ $\quad$ $\sin\theta = \frac{-4}{4} = -1$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$ $\quad$ $\theta = \frac{3\pi}{2}$

$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

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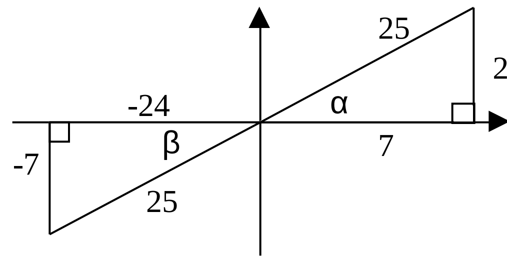
(16 pts.) 4. If $\tan\alpha = \frac{24}{7}$ and $\sin\beta = -\frac{7}{25}$ for a first quadrant angle α and a third quadrant angle β, find and simplify:

$b^2 + 24^2 = 5^2$	$c^2 = 24^2 + 7^2$
$b^2 = 625 - 576$	$c^2 = 576 + 49$
$b = 7$	$a = 25$



(8 pts.) a) $\sin(\alpha + \beta)$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha+\beta) = \left(\frac{24}{25}\right)\left(-\frac{24}{25}\right) + \left(\frac{7}{25}\right)\left(-\frac{7}{25}\right)$$

$$\sin(\alpha+\beta) = \left(-\frac{576}{625}\right) + \left(-\frac{49}{625}\right) = -\frac{625}{625} = -1$$

Answer: -1

(8 pts.) b) $\cos(\alpha - \beta)$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\cos(\alpha-\beta) = \left(\frac{7}{25}\right)\left(-\frac{24}{25}\right) + \left(\frac{24}{25}\right)\left(-\frac{7}{25}\right)$$

$$\cos(\alpha-\beta) = \left(-\frac{168}{625}\right) + \left(-\frac{168}{625}\right) = -\frac{336}{625}$$

Answer: $-\frac{336}{625}$

(12 pts.) 5. Find the exact radian value of the expression whenever it is defined.

(6 pts.) a) $\sin^{-1}\left(-\frac{1}{2}\right)$

Answer: $-\frac{\pi}{6}$

(6 pts.) b) $\tan^{-1}(1)$

Answer: $\frac{\pi}{4}$

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(10 pts.) 6. Write the expression as an algebraic expression in x for $x > 0$.

$\cos\left(\tan^{-1} x\right)$

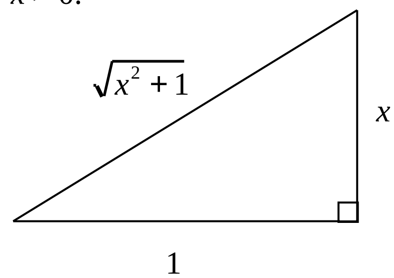
$\alpha = \tan^{-1} x$

$\tan\alpha = x$

$c^2 = x^2 + 1^2$

$c = \sqrt{x^2+1}$

$\cos\alpha = \dfrac{1}{\sqrt{x^2+1}}$



$$\frac{1}{\sqrt{x^2+1}}$$

(12 pts.) 7. Verify the identity:

$\sin(\pi + \theta) = -\sin\theta$

$\sin\pi\cos\theta + \cos\pi\sin\theta =$

$(0)\cos\theta + (-1)\sin\theta =$

$-\sin\theta = -\sin\theta$

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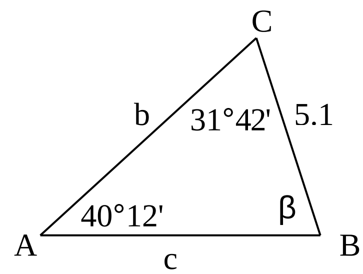
(12 pts.) 8. Solve ΔABC. Round angle measures to the nearest minute and lengths to one decimal place.

$\alpha = 40°12'$, $\gamma = 31°42'$, $a = 5.1$



$$\beta = 180° - (40°12' + 31°42')$$
$$\beta = 180° - (71°54')$$
$$\beta = 108°6'$$

$\beta =$ **108°6'**

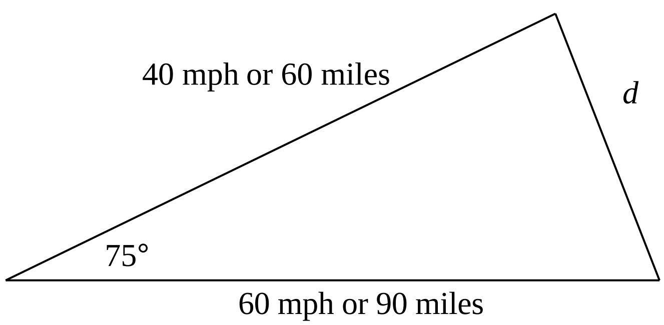
$$\frac{\sin 40°12'}{5.1} = \frac{\sin 108°6'}{b}$$
$$b = \frac{5.1(\sin 108°6')}{\sin 40°12'} = \frac{4.8}{0.65}$$
$$b = 7.5$$

$$\frac{\sin 40°12'}{5.1} = \frac{\sin 31°42'}{c}$$
$$c = \frac{5.1(\sin 31°42')}{\sin 40°12'} = \frac{2.68}{0.65}$$
$$c = 4.2$$

$b =$ **7.5**

$c =$ **4.2**

(12 pts.) 9. Two automobiles leave Lafayette at the same time and travel along straight highways that differ in direction by $75°$. If their speeds are 60 mi./hr. and 40 mi./hr. respectively, how far apart are the cars 1.5 hours after leaving Lafayette? Round your answer to one decimal place. (Draw and label a diagram, set up an equation(s), and solve.)



40(1.5) = 60 miles
60(1.5) = 90 miles

$$d^2 = 90^2 + 60^2 - 2(90)(60)\cos 75°$$
$$d^2 = 8{,}100 + 3{,}600 - 10{,}800(0.2588)$$
$$d^2 = 11{,}700 - 2{,}795.25$$
$$d^2 = 8{,}904.75$$
$$d = 94.4$$

94.4 miles