1. A ladder 10 feet long is leaning against a wall. The foot of the ladder is being pulled away from the wall at 3 feet per second. How fast, in feet per second, is the top of the ladder sliding down the wall when the foot of the ladder is 8 feet from the wall?

   A. 1

   B. 2

   C. 3

   D. 4

   E. 5

2. A spherical tank has radius equal to 10 feet (= 120 inches). Use differentials to estimate, in cubic inches, the amount of paint needed to cover the surface with a layer $\frac{1}{100}$ of an inch thick. ($V = \frac{4}{3}\pi r^3$).

   A. 288π

   B. 480π

   C. 576π

   D. 640π

   E. 960π

3. Find the absolute minimum of the function

   $$f(x) = 4x^3 - 15x^2 + 12x + 7$$

   on the closed interval $[0, 3]$.

   A. 0

   B. 1

   C. 3

   D. 5

   E. 7

4. How many real roots does the equation $x^7 + x + 1 = 0$ have?

   A. 1
   B. 2
   C. 3
   D. 5
   E. 7

5. Find the largest interval on which the function $f(x) = x \sin x + \cos x$, $0 \leq x \leq \pi$, is increasing.

   A. $(0, \pi)$
   B. $(0, \frac{\pi}{2})$
   C. $(\frac{\pi}{2}, \pi)$
   D. $(0, \frac{\pi}{3})$
   E. $(\frac{\pi}{3}, \frac{5\pi}{3})$

6. What is the length of the largest interval on which the function $f(x) = x^3 - 3x^2 - 9x$ is decreasing?

   A. 1
   B. 2
   C. 3
   D. 4
   E. ∞

7. On what interval is the graph of the function

$$f(x) = 1 - \frac{2}{x} + \frac{1}{x^2}$$

concave downward?

A. $(\frac{3}{2}, \infty)$
B. $(1, \frac{3}{2})$
C. $(-\infty, 0)$
D. $(0, 1)$
E. $(1, \infty)$

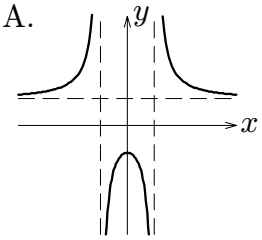
8. $\lim_{x\to\infty} \dfrac{(\ln x)^3}{x^2} =$

A. 0
B. 1
C. $\frac{3}{2}$
D. $\frac{9}{4}$
E. ∞

9. Given the following information about limits, select the graph that could be the graph of $y = f(x)$.

$$\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 0, \qquad \lim_{x\to-1^-} f(x) = \lim_{x\to1^+} f(x) = \infty$$

$$\lim_{x\to-1^+} f(x) = \lim_{x\to1^-} f(x) = -\infty$$

A.


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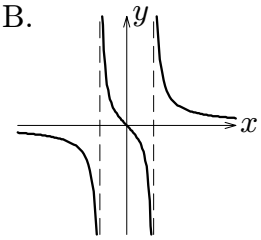

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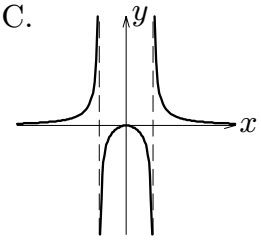

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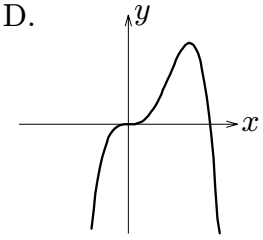

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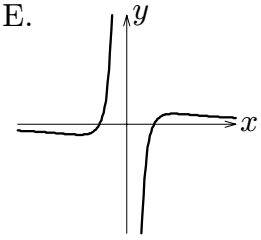


10. The function $f(x) = x^4 - 3x^3 + 3x^2 - x$ has critical numbers $c = \frac{1}{4}, 1$; indeed $f'(x) = (4x-1)(x-1)^2$. At these critical numbers f has

A. a local max. at $\frac{1}{4}$, a local min. at 1

B. a local max. at 1, a local min. at $\frac{1}{4}$

C. a local max. at 1, neither a local max. nor a local min. at $\frac{1}{4}$

D. a local min. at $\frac{1}{4}$, neither a local max. nor a local min. at 1

E. neither a local max. nor a local min. at either $\frac{1}{4}$ or 1

11. Find the maximum value of the function $\dfrac{x^2 + 2x - 4}{x^2}$.

A. $\frac{1}{4}$

B. $\frac{9}{4}$

C. $\frac{7}{4}$

D. $\frac{3}{4}$

E. $\frac{5}{4}$

12. A rectangular cardboard box of 32 in^3 volume with a square base and an open top is to be constructed. Neglecting waste, find the minimum area of cardboard needed.

A. 54 in^2

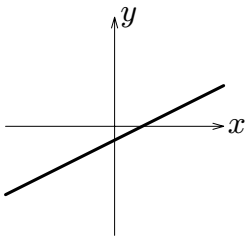
B. 48 in^2

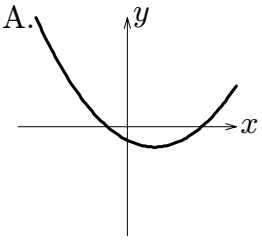
C. 46 in^2

D. 42 in^2

E. 40 in^2

13. Given the graph of $y = f'(x)$ below, select a graph which could be the graph of $y = f(x)$.



A.


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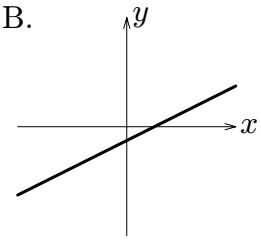

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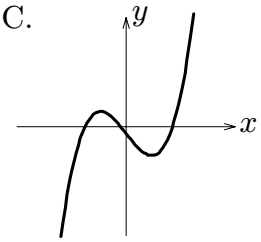

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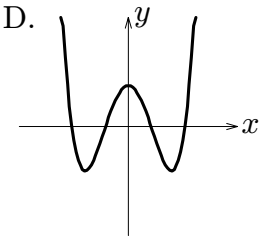

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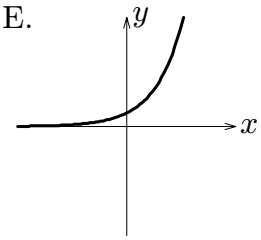

14. If $f''(x) = 12x^2 + 2$, $f(0) = 2$ and $f'(0) = 3$, find $f(1)$.

A. 3

B. 4

C. 5

D. 6

E. 7