1. $\lim_{x\to\infty} \left(\sqrt{x^2+3x} - x\right) =$

A. 0

B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. 3

E. ∞

2. If $f(x) = x\cos(x)$, then $f''\left(\dfrac{\pi}{4}\right) =$

A. $\frac{1}{\sqrt{2}}$

B. $-\frac{\pi}{4}$

C. $\frac{\pi}{4} - \sqrt{2}$

D. $\frac{\pi}{4}(1-\sqrt{2})$

E. $-\frac{1}{\sqrt{2}}\left(\frac{\pi}{4}+2\right)$

3. If $f(x) = \dfrac{x}{\sin x}$, then $f'\left(\dfrac{\pi}{4}\right) =$

A. $1 - \frac{\pi}{2}$

B. $\sqrt{2}\left(1 - \frac{\pi}{4}\right)$

C. $\sqrt{2}\left(1 + \frac{\pi}{4}\right)$

D. $\sqrt{2}\left(1 + \frac{\pi}{2}\right)$

E. $1 + \frac{\pi}{4}$

4. Let $g(x) = f(f(x))$ and $f(1) = 2$, $f(2) = -1$, $f'(2) = 7$, $f'(1) = 5$, $f'(-1) = 4$, $f'(4) = 9$, $f'(7) = 3$. Then $g'(1) =$

A. 35

B. 63

C. 180

D. 189

E. 243

5. If $f(x) = \sqrt{x + \sqrt{x}}$, then $f'(1) =$

A. $\frac{1}{2\sqrt{2}}$

B. $\frac{1}{\sqrt{2}}$

C. $\sqrt{2}$

D. $2\sqrt{2}$

E. $\frac{3}{4\sqrt{2}}$

6. If $x^2 - xy + y^3 = 14$ then $\dfrac{dy}{dx} =$

A. $-\dfrac{2xy}{x + 3y^2}$

B. $-\dfrac{3x^2 + y}{2x - y}$

C. $\dfrac{y - 2x}{3y^2 - x}$

D. $\dfrac{x + y}{x^2 + 2y}$

E. $\dfrac{xy}{x^2 - y^2}$

7. The function $f(x)$ has derivative $f'(x) = x(x+1)^3(x-1)^2$. Consider the following statements

   I. f has a local maximum at $x = -1$

   II. f has a local minimum at $x = -1$

   III. f has a local maximum at $x = 0$

   IV. f has a local minimum at $x = 0$

   V. f has a local maximum at $x = 1$

   VI. f has a local minimum at $x = 1$

   A. I, III, VI are true

   B. I and IV are true, V and VI are false

   C. I and V are true, III and VI are false

   D. III and VI are true, I and II are false

   E. I, IV and VI are true

8. Find the absolute maximum of the function $f(x) = x^3 - x^2 - x$ on the interval $-10 \leq x \leq 1$.

   A. $-\frac{2}{9}$

   B. $\frac{5}{27}$

   C. -1

   D. $\frac{7}{27}$

   E. $\frac{2}{9}$

9. What is the length of the longest interval on which the function $f(x) = \dfrac{x}{x^2+1}$ is increasing?

   A. 0

   B. 1

   C. 2

   D. 4

   E. ∞

10. Determine where the function $f(x) = x + \dfrac{1}{x^2}$ is concave upward.

A. $(-\infty, 0)$ and $(0, \infty)$

B. $(-1, 0)$

C. $(0, \infty)$

D. $(0, 1)$

E. nowhere

11. Given the function $f(x) = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$, consider the following statements

I. $y = 1$ is a horizontal asymptote of f

II. $y = -1$ is a horizontal asymptote of f

III. $x = 0$ is a vertical asymptote of f

A. I, II, III are false

B. I is true, II and III are false

C. I and II are true, III is false

D. I and III are true, II is false

E. I, II and III are true

12. Consider the statements

I. $\lim_{x \to 0^+} \ln |x| = -\infty$

II. $\lim_{x \to 0^-} \ln |x| = \infty$

III. $\lim_{x \to -\infty} \ln |x| = -\infty$

A. I, II, III are false

B. I is true, II and III are false

C. I and II are true, III is false

D. I and III are true, II is false

E. I, II and III are true

13. $\lim\limits_{x\to\infty} \cos\left(\dfrac{1}{x}\right) =$

A. -1

B. 0

C. 1

D. $-\infty$

E. does not exist

14. $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2} =$

A. 0

B. $\frac{1}{2}$

C. 1

D. 2

E. ∞

15. A colony of bacteria, undergoing exponential growth, starts with 200 bacteria. One hour later it contains 400 bacteria. How many hours does it take to reach 2000 bacteria?

A. 5

B. $\ln 1600$

C. $\ln 200$

D. $\dfrac{\ln 10}{\ln 2}$

E. $\ln 10$

16. $\sin(\tan^{-1} x) =$

A. $\dfrac{1}{1+x^2}$

B. $\sqrt{1-x^2}$

C. $\dfrac{1-x^2}{1+x^2}$

D. $\dfrac{x}{\sqrt{1+x^2}}$

E. $1+x^2$

17. If $f(a) = b$ and $f'(a) = c$, use differentials to approximate $f(a + \frac{1}{2}) - f(a)$.

A. $c - b$

B. $\frac{c}{2}$

C. b

D. $\frac{b}{2c}$

E. bc

18. If $f''(x) = 20x^3 - 6x + 2$, $f'(1) = 2$ and $f(1) = 4$, then $f(-1) =$

A. -1

B. 0

C. 2

D. 4

E. 8

19. $\dfrac{d}{dx} \displaystyle\int_1^x \sinh(t^2)dt =$

A. $\sinh(x)$

B. $\sinh(x^2)$

C. $2x\cosh(x^2)$

D. $2x\sinh(x^2)$

E. $\cosh(x)$

20. Let $F(x) = \int_0^{x^2} \sin(t^2)dt$. Consider the following statements

I. $F(0) = 0$

II. $F(0) < F(1)$

III. F is increasing for all values of x

IV. $F(-1) = -F(1)$

A. I, II, III, IV are true

B. I, II, III are true, IV is false

C. I, II are true, III, IV are false

D. I is true, II, III, IV are false

E. I, II, III, IV are false

21. $\int \frac{x}{1+4x^2}\,dx =$

A. $\frac{x^2}{1+4x^2} + C$

B. $\frac{1}{2(1+4x^2)^2} + C$

C. $\frac{1}{1+4x^2} + C$

D. $\frac{1}{4}\ln(1+4x^2)+C$

E. $\frac{1}{8}\ln(1+4x^2)+C$

22. $\int_1^2 (1+3x^2+x^3)dx =$

A. $11\frac{3}{4}$

B. $32\frac{3}{4}$

C. 42

D. 57

E. 83

23. $\int_1^2 (1 + \frac{1}{x} + \frac{1}{x^2})\, dx =$

A. $-\frac{1}{4}$

B. $\frac{3}{2}$

C. $\ln 2 + \frac{3}{2}$

D. $\ln 2 - \frac{1}{2}$

E. $\ln 2 + 1$

24. The length of a rectangle is decreasing at a rate of 1 foot per second, but the area remains constant. At what rate, in feet per second, is the rectangle's width increasing when its length is 10 feet and its width is 5 feet?

A. $\frac{1}{10}$

B. $\frac{1}{5}$

C. $\frac{1}{4}$

D. $\frac{1}{2}$

E. 1

25. Find the shortest distance from the point $(1, 4)$ to the parabola $y^2 = 2x$.

A. $\sqrt{6}$

B. $\sqrt{5}$

C. 2

D. $\sqrt{3}$

E. $\sqrt{2}$