MA 161 FINAL EXAM **APRIL 30, 2001**

Name Student ID number

Lecturer Recitation Instructor

Instructions:

1. This package contains 25 problems, each worth 8 points, for a total of 200 points.
2. Please supply all information requested above and on the mark–sense sheet.
3. Work only in the space provided, or on the backside of the pages. Mark your answers clearly on the scantron. Also circle your choice for each problem in this booklet.
4. No books, notes, or calculator, please.

1. Solve the inequality $x^3 \geq 2x^2 + 3x$

A. $x \geq 3$

B. $-1 \leq x \leq 0$ or $x \geq 3$

C. $x \leq -1$ or $0 \leq x \leq 3$

D. $-3 \leq x \leq 0$ or $x \geq 1$

E. $x \leq -3$ or $0 \leq x \leq 1$

2. $\sin(\cos^{-1} x) =$

A. $\tan x$

B. $\dfrac{x}{\sqrt{1-x^2}}$

C. $\dfrac{\sqrt{1-x^2}}{x}$

D. $\dfrac{1}{\sqrt{1-x^2}}$

E. $\sqrt{1-x^2}$

3. If $f(x) = \dfrac{1}{2-x}$ and $g(x) = \dfrac{x}{x-3}$ then the domain of $f \circ g$ is

A. $x = \dfrac{5}{3}, 2, 3$

B. $x \neq \dfrac{5}{3}, 2$

C. $x \neq 6$

D. $x \neq 3, 6$

E. $x \neq 2, 3, 6$

4. Express the quantity $x + \dfrac{\ln x}{2} - 2\ln(x+1)$ as a single logarithm.

A. $\ln \dfrac{x\sqrt{x}}{(x+1)^2}$

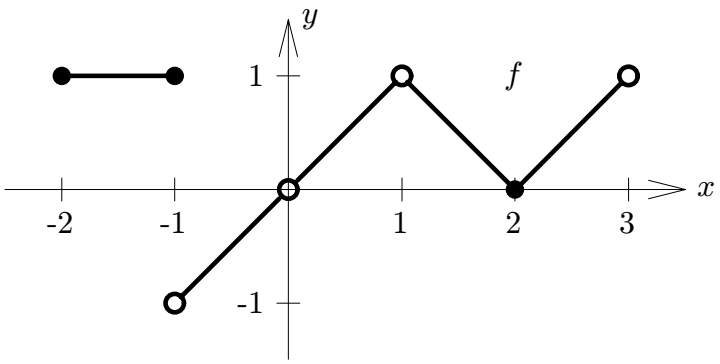
B. $\ln(\sqrt{x}(x+1)^2)$

C. $\ln(x+\sqrt{x}-(x+1)^2)$

D. $\ln \dfrac{x}{4(x+1)}$

E. $\ln \dfrac{e^x\sqrt{x}}{(x+1)^2}$

5. Given the following graph of the function f, which statement is true?



A. $\lim_{x\to 0} f(x)$ does not exist

B. $f(-1)$ does not exist

C. $\lim_{x\to 1} f(x) = f(3)$

D. $\lim_{x\to 1} f(x) = 1$

E. $\lim_{x\to -1} f(x) = -1$

6. Find the number a such that $\lim_{x\to 1} \dfrac{x^2 + 3ax + a + 3}{x^2 + 2x - 3}$ exists and is finite.

A. $a = -2$

B. $a = -1$

C. $a = 0$

D. $a = 1$

E. $a = 2$

7. The tangent line to $y = f(x)$ at $(3, -5)$ passes through the point $(-2, 4)$. Find $f'(3)/f(3)$.

A. $\frac{9}{10}$

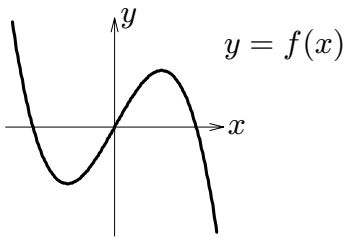
B. $\frac{9}{15}$

C. $-\frac{9}{15}$

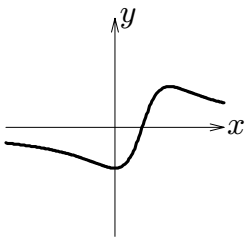
D. $\frac{9}{25}$

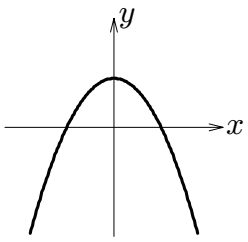
E. $-\frac{18}{25}$

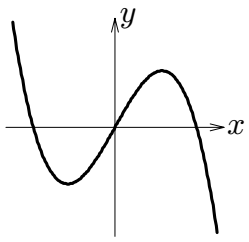
8. Given the graph of the function f

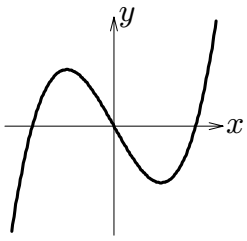


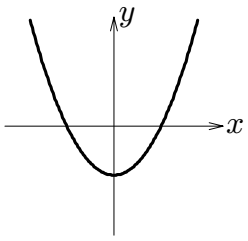
the graph of its derivative looks most like

A.


B.


C.


D.


E.


9. $\lim_{h\to 0} \dfrac{e^h - 1}{h} =$

A. 1

B. 0

C. e

D. -1

E. none of the above

10. If $g(x) = \dfrac{e^x}{x + e^x}$ then $g'(x) =$

A. $\dfrac{e^{2x}(x-1)}{(x+e^x)^2}$

B. $\dfrac{e^x(x-1)}{(x+e^x)^2}$

C. $\dfrac{e^x}{1+e^x}$

D. $\dfrac{2e^{2x} + (x-1)e^x}{(x+e^x)^2}$

E. $\dfrac{1}{(x+1)^2}$

11. The frequency of vibrations of a vibrating violin string is $f = \dfrac{1}{2L}\sqrt{\dfrac{T}{\rho}}$, where L is the length, T the tension, and ρ the linear density. The rate of change of the frequency with respect to the tension is

A. $\dfrac{1}{4L\sqrt{\rho T}}$

B. $\dfrac{1}{4L\sqrt{\dfrac{T}{\rho}}}$

C. $-\dfrac{1}{2L^2}\sqrt{\dfrac{T}{\rho}}$

D. $-\dfrac{1}{4\rho L}\sqrt{\dfrac{T}{\rho}}$

E. $\dfrac{1}{2L\sqrt{\rho}}$

12. The slope of the tangent to the curve $y = (2 + \sin x)\cos x$ at $\left(\dfrac{5\pi}{4}, \dfrac{1}{2} - \sqrt{2}\right)$ is

A. 0

B. $\dfrac{1}{2}$

C. $\dfrac{1}{\sqrt{2}}$

D. $\sqrt{2}$

E. -1

13. $\dfrac{d}{dx}\, e^{\sqrt{x}} =$

A. $e^{\sqrt{x}}$

B. $\dfrac{e^{\sqrt{x}}}{2}$

C. $e^{\frac{1}{2\sqrt{x}}}$

D. $\dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$

E. none of the above

14. If $x^2 - y^2 = 1$ then $\dfrac{dy}{dx} =$

A. $-\dfrac{y}{x^2}$

B. $\dfrac{x}{y}$

C. $-\dfrac{x}{y}$

D. $\dfrac{y}{x}$

E. $\dfrac{y^2}{x}$

15. A kite 8 m above the ground flies horizontally at a speed of 2 m/s. Assume our end of the rope is on ground level. When 10 m of string have been let out, the angle between the string and the horizontal is decreasing at a rate of (in rad/s)

A. $\frac{1}{25}$

B. $\frac{4}{25}$

C. $\frac{3}{100}$

D. $\frac{1}{10}$

E. $\frac{16}{225}$

16. The function $f(x) = x^2(6 - x)$, $0 \leq x \leq 6$, attains its absolute maximum when $x =$

A. 0

B. 1

C. 2

D. 4

E. 6

17. Suppose $h(x)$ is differentiable and $h'(x) \geq 3$ for all real numbers x; furthermore $h(0) = 0$. Which of the following statements is(are) true?

    I. $h(2) \leq 4$

    II. $h(2) \geq 6$

    III. The equation $h(x) = 0$ has a unique root on $[0, 2]$

    A. only I

    B. only II

    C. only III

    D. only I and III

    E. only II and III

18. On which one of the following intervals is the function $x^4 - \dfrac{3x^2}{2}$ is concave downward:

    A. $\left(-\sqrt{\dfrac{3}{2}}, \sqrt{\dfrac{3}{2}}\right)$

    B. $\left(0, \dfrac{\sqrt{3}}{2}\right)$

    C. $\left(\dfrac{\sqrt{3}}{2}, \infty\right)$

    D. $\left(-\dfrac{1}{2}, \dfrac{1}{2}\right)$

    E. $\left(\dfrac{1}{2}, \infty\right)$

19. $\lim_{x \to 0^+} (1+x)^{2/x} =$

A. 0

B. 1

C. ∞

D. e

E. e^2

20. A farmer, versed in calculus, wants to fence a rectangular area of 100 m^2, then divide it in two by a fence running along a diagonal. He denotes the length of one side by x. To find the shortest fence needed for this purpose, he will have to find the minimum of the function

A. $x(100-x)+\sqrt{x^2+(100-x)^2}$

B. $4x(100-x)+\sqrt{x^2+(100-x)^2}$

C. $2x+\dfrac{200}{x}+\sqrt{x^2+\dfrac{10,000}{x^2}}$

D. $100x+\sqrt{x^2+\dfrac{100}{x^2}}$

E. $x(100-x)+\sqrt{x^2+\dfrac{100}{x^2}}$

21. If $f'(x) = 3\sqrt{x} - 1$ and $f(4) = 0$ then $f(9) =$

A. 7

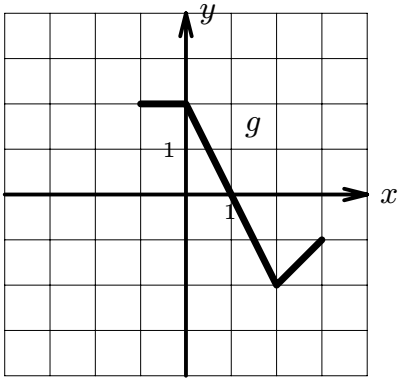
B. 11

C. 27

D. 33

E. 47

22. If g is given by its graph, $\int_{-1}^{3} g(x)dx =$



A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. 0

D. 1

E. -1

23. $\int_{-\frac{1}{2}}^{0} (2x+1)^8 dx =$

A. 0

B. $\frac{1}{18}$

C. $-\frac{1}{16}$

D. $\frac{1}{8}$

E. $-\frac{1}{4}$

24. $\int_{0}^{\frac{\pi}{3}} 2\cos u du =$

A. 1

B. -1

C. $\sqrt{3}$

D. $1-\sqrt{3}$

E. 2

25. If $h(x) = \displaystyle\int_0^{\sqrt{x}} e^{t^2}\,dt$ then $h'(\ln 2) =$

A. 2

B. e^2

C. $\dfrac{1}{\sqrt{\ln 2}}$

D. $e^{\sqrt{2}}$

E. $2e \ln 2$