1. The distance of the point $(2, 1, 3)$ to the xy plane is

A. $\sqrt{5}$

B. $\sqrt{10}$

C. $\sqrt{13}$

D. 1

E. 3

2. If $(2\mathbf{i} + \mathbf{j} - 3\mathbf{k}) \times (4\mathbf{i} + 2\mathbf{j} - 6\mathbf{k}) = c_1\mathbf{i} + c_2\mathbf{j} + c_3\mathbf{k}$ then $c_1 =$

A. 0

B. 1

C. -2

D. 4

E. -8

3. The area between the curves $y = \dfrac{\cos x}{2}$ and $y = \dfrac{\sin x}{2}$, $0 \le x \le \dfrac{\pi}{2}$ is

A. 0

B. 1

C. $\dfrac{\pi}{4}$

D. $\sqrt{2} - 1$

E. $\dfrac{\sqrt{3}}{2} + 1$

4. A solid lies between the planes $z = 1$ and $z = 3$. If we intersect it with a plane parallel to the xy plane, at distance z to this latter, the cross section is a circle of radius $\sqrt{z}$, $1 \le z \le 3$. Find the volume of the solid.

A. π

B. 2π

C. 3π

D. 4π

E. 6π

5. The region between the curves $y = \dfrac{1}{x^2 + x}$, $y = 0$, $x = 1$, and $x = 2$ is revolved about the y axis. The volume of the solid generated is

   A. $\dfrac{3\pi}{5}$

   B. $\pi \ln 2$

   C. $2\pi \ln \dfrac{3}{2}$

   D. $\dfrac{5\pi}{6}$

   E. $\dfrac{5\pi \ln 3}{\ln 2}$

6. It takes 3J of work to stretch a spring by 5 cm beyond its natural length. How much work is needed to stretch the same spring by 10 cm (beyond natural length)?

   A. 12J

   B. 9J

   C. 8J

   D. 6J

   E. 4J

7. $\int_0^\infty xe^{-2x}dx =$

A. $\frac{1}{4}$

B. $\frac{1}{e}$

C. 1

D. 2

E. e

8. $\int_0^\pi 2\sin^2 x\,dx =$

A. 0

B. $\frac{1}{4}$

C. 1

D. $\frac{\pi}{2}$

E. π

9. Evaluation of $\displaystyle\int \frac{\sqrt{4+x^2}}{x}\,dx$ leads to the following integral:

A. $\displaystyle 2\int \sin^3\theta\,d\theta$

B. $\displaystyle 2\int \frac{\sec^3\theta}{\tan\theta}\,d\theta$

C. $\displaystyle \int \frac{d\theta}{\cos^2\theta}$

D. $\displaystyle \frac{1}{2}\int \tan^2\theta\,d\theta$

E. $\displaystyle 2\int \sec\theta\tan^2\theta\,d\theta$

10. The partial fraction decomposition of $\dfrac{x+1}{x^3-2x^2+x}$ is of the form

A. $\dfrac{A}{x}+\dfrac{B}{x-1}+\dfrac{C}{(x-1)^2}$

B. $\dfrac{A}{x^3}+\dfrac{B}{x^2}+\dfrac{C}{x}$

C. $\dfrac{A}{x^2+1}+\dfrac{Bx+C}{x^2}$

D. $\dfrac{A}{x^2}+\dfrac{B}{x}+\dfrac{C}{x-1}$

E. $\dfrac{Ax+B}{x^2+1}+\dfrac{C}{x}$

11. The improper integrals I. $\displaystyle\int_2^\infty \frac{dx}{x^2}$ and II. $\displaystyle\int_0^{\sqrt{2}} \frac{dx}{\sqrt{x}}$

A. both converge

B. only I. converges

C. only II. converges

D. neither converges

E. none of the above is true

12. Which is/are true?

I. If $\displaystyle\sum_{n=1}^{\infty} a_n$ absolutely converges then $\displaystyle\lim_{n\to\infty} a_n = 0$.

II. If $\displaystyle\sum_{n=1}^{\infty} a_n$ absolutely converges then $\displaystyle\sum_{n=1}^{\infty} a_n$ converges.

III. $\displaystyle\int_1^\infty \frac{dx}{x^2} \le \sum_{n=1}^{\infty} \frac{1}{n^2}$.

A. only I. is true

B. only II. is true

C. only I. and II are true

D. only II. and III. are true

E. All three are true

13. $\lim_{k\to\infty} \dfrac{2\sqrt[3]{k}+k^2}{3\sqrt{k}+k^3} =$

A. 0

B. $\frac{2}{3}$

C. 1

D. $\frac{3}{2}$

E. ∞

14. The series $\sum_{n=3}^{\infty} \dfrac{1}{n^{3/2}\ln n}$

A. converges by the comparison test, using $\sum_{n=3}^{\infty} \frac{1}{n^{3/2}}$

B. converges by the root test

C. diverges by the ratio test

D. diverges by the comparison test, using $\sum_{n=3}^{\infty} \frac{1}{n^{3/2}}$

E. diverges by the limit comparison test, using $\sum_{n=3}^{\infty} \frac{1}{n^{3/2}}$

15. If the ratio test is used for the series $\sum_{n=1}^{\infty} \frac{2^n n!}{(2n)!}$ then $\lim_{n\to\infty} \frac{a_{n+1}}{a_n}$ equals

A. 0

B. $\frac{1}{2}$

C. 1

D. 2

E. ∞

16. Which is/are conditionally convergent?

I. $\sum_{1}^{\infty} \frac{(-1)^n}{n+2}$ II. $\sum_{1}^{\infty} \frac{(-1)^n n}{n+3}$ III. $\sum_{1}^{\infty} \frac{(-1)^n}{2^n+1}$

A. only I and II

B. only I and III

C. only I

D. only II and III

E. only III

17. The interval of convergence of $\sum_{k=0}^{\infty} \frac{(k+1)x^k}{2^k}$ is

A. $\left[-\frac{1}{2}, \frac{1}{2}\right]$

B. $\left(-\frac{1}{2}, \frac{1}{2}\right)$

C. $[-2, 2]$

D. $[-2, 2)$

E. $(-2, 2)$

18. The first three terms of the power series of $\frac{x}{1-2x^2}$ are

A. $2x - 4x^2 - 8x^3$

B. $x + 2x^3 + 4x^5$

C. $x - 2x^3 + 4x^5$

D. $2x^3 - 4x^5 + 8x^7$

E. $2x + 2x^3 + 2x^5$

19. The Maclaurin series of $\sqrt{1+2x}$ is

A. $1+x-\dfrac{x^2}{4}\pm\ldots$

B. $1+\dfrac{x}{2}-\dfrac{x^2}{8}\pm\ldots$

C. $1+x-\dfrac{x^2}{2}\pm\ldots$

D. $1+x-x^2\pm\ldots$

E. $1+\dfrac{x}{2}-\dfrac{x^2}{4}\pm\ldots$

20. The parametric equations $x=1+4t^2$, $y=1+2t$ can be rewritten as the Cartesian equation

A. $x=2+2y+y^2$

B. $x-2y=1$

C. $x+2y=2$

D. $x=2-2y+y^2$

E. $x=1+y^2$

21. Find the tangent to the curve $x = 2 - t$, $y = t - t^2$ at the point where $t = 2$.

A. $y = -3x+2$

B. $y = -x + 1$

C. $y = -x - 2$

D. $y = -3x - 1$

E. $y = 3x - 2$

22. The length of the curve $x = \dfrac{2t^3}{3}$, $y = \dfrac{t^4}{4} - \dfrac{t^2}{2}$, $1 \leq t \leq 2$, is

A. $\dfrac{7}{4}$

B. $\dfrac{9}{4}$

C. $\dfrac{21}{4}$

D. 2

E. 3

23. Which curve is described by the polar equation $r = \frac{1}{2} + \sin\theta$?

24. The area of one loop of the curve $r = \cos 4\theta$ is

A. $\frac{\pi}{16}$

B. $\frac{\pi}{8}$

C. $\frac{\pi}{16} + \frac{\sqrt{2}}{16}$

D. $\frac{\pi}{8} + \frac{\sqrt{2}}{16}$

E. $\frac{\pi}{32}$

25. The asymptotes of the hyperbola $4x^2 - 4x - y^2 = 3$ are

A. $y = 1 \pm 2x$

B. $y = \frac{1}{2} \pm \frac{x}{2}$

C. $y = \frac{3}{2} \pm \frac{x}{2}$

D. $y = \pm(2x - 1)$

E. $y = \pm\frac{x - 2}{2}$