

MA 162

Exam 2

Spring 2000

NAME SOLUTION KEY

STUDENT ID _____

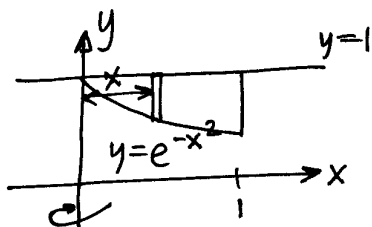
REC. INSTR. _____ REC. TIME. _____

INSTRUCTOR _____

INSTRUCTIONS:

1. Make sure that you have all 7 test pages.
 2. Fill in the information requested above and on the answer sheet.
 3. Mark the letter of your response for each question on the mark-sense answer sheet.
 4. There are 10 problems worth 9 points each, and 2 worth 5 points each for a total of 100 points.
 5. No books or notes or calculators may be used.
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1. Suppose R is the region between the graphs of $y = e^{-x^2}$ and $y = 1$, above the interval $0 \leq x \leq 1$. Find the volume of the solid obtained by revolving R about the y axis.



Shell Method:

$$V = \int_0^1 2\pi x (1 - e^{-x^2}) dx$$

$$= 2\pi \int_0^1 (x - xe^{-x^2}) dx = 2\pi \left(\frac{x^2}{2} + \frac{e^{-x^2}}{2} \right) \Big|_0^1$$

$$= 2\pi \left(\frac{1}{2} + \frac{e^{-1}}{2} \right) - 2\pi \left(0 + \frac{1}{2} \right) = \frac{\pi}{e}$$

A. $2\pi - \frac{\pi}{e}$

B. $\pi + \frac{\pi}{e}$

C. $\pi - \frac{\pi}{e}$

D. $\frac{\pi}{e}$

E. $\pi + \frac{2\pi}{e}$

2. Find the length of the curve $y = 2x^{\frac{3}{2}}$ for $0 \leq x \leq \frac{1}{3}$.

$$y' = 3x^{\frac{1}{2}}$$

$$L = \int_0^{\frac{1}{3}} \sqrt{1 + (3x^{\frac{1}{2}})^2} dx$$

$$= \int_0^{\frac{1}{3}} \sqrt{1 + 9x} dx$$

$$= \frac{1}{9} \cdot \frac{2}{3} (1 + 9x)^{\frac{3}{2}} \Big|_0^{\frac{1}{3}}$$

$$= \frac{2}{27} \left(4^{\frac{3}{2}} - 1^{\frac{3}{2}} \right) = \frac{2}{27} (7) = \frac{14}{27}$$

A. $\frac{7}{27}$

B. $\frac{7}{9}$

C. $\frac{14}{27}$

D. $\frac{2}{9}$

E. $\frac{16}{27}$

3. Suppose that a force of 2 lbs is needed to stretch a spring $\frac{1}{2}$ ft. beyond its natural length. Calculate the work required to stretch it an additional $\frac{1}{2}$ ft.

Hooke's Law: $F = kx$, $2 = k \frac{1}{2} \rightarrow k = 4$ (A) $\frac{3}{2}$ ft-lbs

$$\text{Work} = \int_{0.5}^1 4x \, dx = 2x^2 \Big|_{0.5}^1$$

$$= 2(1 - 0.25) = 2(0.75) = 1.5$$

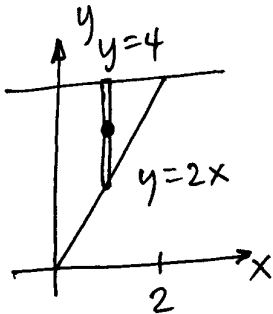
B. 2 ft-lbs

C. $\frac{1}{2}$ ft-lbs

D. 1 ft-lb

E. 3 ft-lbs

4. A plate occupies the part of the first quadrant between the lines $y = 2x$ and $y = 4$. Find the x coordinate of center of gravity.



$$\bar{x} = \frac{M_{x=0}}{\text{mass}} = \frac{M_y}{\text{mass}} = \frac{\frac{8}{3}}{4} = \frac{2}{3}$$

(see below)

A. 1

(B) $\frac{2}{3}$ C. $\frac{4}{3}$ D. $\frac{3}{2}$ E. $\frac{1}{2}$

$$\text{mass} = \int_0^2 \rho(4-2x) \, dx = \rho(4x - x^2) \Big|_0^2 = \rho(4)$$

$$\begin{aligned} M_{x=0} &= \int_0^2 x \rho(4-2x) \, dx = \int_0^2 \rho(4x - 2x^2) \, dx \\ &= \rho(2x^2 - \frac{2}{3}x^3) \Big|_0^2 = \rho(8 - \frac{2}{3}(8)) = \rho(\frac{8}{3}) \end{aligned}$$

5. If the third Taylor polynomial of a given function $f(x)$ is $p_3(x) = 1 - x + \frac{x^2}{3} + \frac{x^3}{2}$ then the values of $f''(0)$ and $f'''(0)$ are, respectively

Recall: $p_3(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3$ (A) $\frac{2}{3}$ and 3

B. $\frac{1}{3}$ and $\frac{1}{2}$

C. $\frac{2}{3}$ and $\frac{1}{2}$

D. $\frac{1}{6}$ and 3

E. both are 3

Therefore: $\frac{f''(0)}{2!}x^2 = \frac{1}{3}x^2 \rightarrow \frac{f''(0)}{2} = \frac{1}{3}$

$\rightarrow f''(0) = \frac{2}{3}$

and $\frac{f'''(0)}{3!}x^3 = \frac{1}{2}x^3 \rightarrow \frac{f'''(0)}{3!} = \frac{1}{2}$

$\rightarrow f'''(0) = \frac{3 \cdot 2}{2} = 3$

6. $\lim_{m \rightarrow \infty} \frac{\sqrt{m^2 + m - 1}}{2m - 1} =$

$= \lim_{m \rightarrow \infty} \left(\frac{\sqrt{m^2 + m - 1}}{2m - 1} \right) \left(\frac{\frac{1}{m}}{\frac{1}{m}} \right)$

$= \lim_{m \rightarrow \infty} \left(\frac{\sqrt{\frac{m^2 + m - 1}{m^2}}}{\frac{2m - 1}{m}} \right)$

$= \lim_{m \rightarrow \infty} \frac{\sqrt{1 + \frac{1}{m} - \frac{1}{m^2}}}{2 - \frac{1}{m}} = \frac{\sqrt{1}}{2} = \frac{1}{2}$

A. ∞

B. 2

C. 1

(D) $\frac{1}{2}$

E. 0

$$7. \lim_{k \rightarrow \infty} (\sqrt{k+5} - \sqrt{k}) = \infty - \infty$$

$$= \lim_{k \rightarrow \infty} (\sqrt{k+5} - \sqrt{k}) \left(\frac{\sqrt{k+5} + \sqrt{k}}{\sqrt{k+5} + \sqrt{k}} \right)$$

$$= \lim_{k \rightarrow \infty} \frac{k+5 - k}{\sqrt{k+5} + \sqrt{k}} = \lim_{k \rightarrow \infty} \frac{5}{\sqrt{k+5} + \sqrt{k}} = 0$$

- A. $-\frac{1}{2}$
 B. 0
 C. 1
 D. $\sqrt{5}$
 E. ∞

$$8. \sum_{n=0}^{\infty} \frac{3^{n-2}}{4^{n+1}} = \sum_{n=0}^{\infty} \frac{3^{-2} \cdot 3^n}{4 \cdot 4^n}$$

$$= \sum_{n=0}^{\infty} \frac{1}{36} \left(\frac{3}{4} \right)^n$$

$$= \frac{1}{36} \left(\frac{1}{1 - \frac{3}{4}} \right)$$

$$= \frac{1}{36} (4) = \frac{1}{9}$$

- A. $\frac{1}{4}$
 B. $\frac{1}{9}$
 C. $\frac{3}{16}$
 D. 1
 E. the series diverges

9. The series $\sum_{n=1}^{\infty} \frac{1}{2n - \sqrt{n}}$

$$\sum \frac{1}{n} \text{ diverges}$$

$$\text{and } \lim_{n \rightarrow \infty} \frac{\frac{1}{n}}{\frac{1}{2n - \sqrt{n}}}$$

$$= \lim_{n \rightarrow \infty} \frac{2n - \sqrt{n}}{n} = 2$$

Therefore $\sum \frac{1}{n}$ and $\sum \frac{1}{2n - \sqrt{n}}$ both diverge by limit comparison test.

- A. converges by the integral test
- B. converges by the limit comparison test
- C. converges by the alternating series test
- D. diverges by the ratio test
- E. diverges by the limit comparison test

10. Which of the following series converges?

I. $\sum_{n=1}^{\infty} (-1)^n \frac{n+1}{n}$; diverges

II. $\sum_{n=0}^{\infty} \frac{2^n}{n!}$; converges

III. $\sum_{n=0}^{\infty} e^n$ diverges.

- A. only I
- B. only II
- C. only III
- D. only I and II
- E. none of the series converge

I. and II. $\lim_{n \rightarrow \infty} a_n \neq 0$.

II. Converges by Ratio Test

$$a_{n+1} \cdot \frac{1}{a_n} = \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n} = 2 \cdot \frac{1}{n+1} \Rightarrow 0 \text{ as } n \rightarrow \infty$$

11. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1}$

$\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1}$ converges by Alternating

Series Test. ($a_n \rightarrow 0$ and $|a_n|$ are decreasing.)

A. diverges

B. converges absolutely

C. converges conditionally

However $\sum_{n=1}^{\infty} \frac{1}{2n+1}$ diverges (use limit comparison with divergent $\sum_{n=1}^{\infty} \frac{1}{n}$)

Therefore $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1}$ converges conditionally

12. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\sqrt{n}}$

converges absolutely since

$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ is a convergent p -series.

A. diverges

B. converges absolutely

C. converges conditionally