

NAME SOLUTION KEY

STUDENT ID _____

REC. INSTR. _____ REC. TIME. _____

INSTRUCTOR _____

INSTRUCTIONS:

1. Make sure that you have all 7 test pages.
 2. Fill in the information requested above and on the answer sheet.
 3. Mark the letter of your response for each question on the mark-sense answer sheet.
 4. There are 11 problems worth 9 points each.
 5. No books or notes or calculators may be used.
-

Some infinite series:

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$$

$$(1+x)^s = \sum_{n=0}^{\infty} \binom{s}{n} x^n$$

1. For which x does the series $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n^2 3^{n+1}}$ converge?
use Ratio Test:

$$\left| a_{n+1} \cdot \frac{1}{a_n} \right| = \left| \frac{x^{n+1}}{(n+1)^2 3^{n+2}} \cdot \frac{n^2 3^{n+1}}{x^n} \right|$$

$$= \frac{n^2}{(n+1)^2} \cdot \frac{1}{3} \cdot |x| \rightarrow \frac{1}{3} |x| \text{ as } n \rightarrow \infty$$

A. $-\frac{1}{3} < x < \frac{1}{3}$

B. $-\frac{1}{3} \leq x \leq \frac{1}{3}$

C. $-3 \leq x < 3$

D. $-3 < x \leq 3$

E. $-3 \leq x \leq 3$

Series converges for $\frac{1}{3} |x| < 1 \rightarrow |x| < 3$
 $\rightarrow -3 < x < 3$

Endpoints: $x=3 \rightarrow \sum_{n=1}^{\infty} \frac{(-1)^n (3)^n}{n^2 3^{n+1}} = \sum_{n=1}^{\infty} (-1)^n \cdot \frac{1}{3} \cdot \frac{1}{n^2}$ converges

$x=-3 \rightarrow \sum_{n=1}^{\infty} \frac{(-1)^n (-3)^n}{n^2 3^{n+1}} = \sum_{n=1}^{\infty} \frac{1}{3} \cdot \frac{1}{n^2}$ converges

2. If the power series of $f(x)$ is $\sum_{n=1}^{\infty} \frac{2^{n-1} x^n}{n+2}$, what is $f^{(4)}(0)$?

$$\frac{f^{(4)}(0)}{4!} = \frac{2^{4-1}}{4+2}$$

A. $\frac{2}{9}$

B. $\frac{4}{3}$

C. 32

D. $\frac{1}{3}$

E. $\frac{16}{3}$

$$\rightarrow f^{(4)}(0) = \frac{2^3}{6} \cdot 4!$$

$$= \frac{2^3 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6} = 32$$

3. Find the first three nonzero terms for the power series of $f(x) = \frac{x}{1+3x^2}$.

Start with geometric series

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots, \quad |x| < 1$$

$$\rightarrow \frac{1}{1+x} = \frac{1}{1-(-x)} = 1 + (-x) + (-x)^2 + (-x)^3 + \dots, \quad |x| < 1$$

$$= 1 - x + x^2 - x^3 + \dots, \quad |x| < 1$$

$$\rightarrow \frac{1}{1+3x^2} = 1 - (3x^2) + (3x^2)^2 - (3x^2)^3 + \dots, \quad |3x^2| < 1$$

$$\rightarrow \frac{x}{1+3x^2} = x - 3x^3 + 9x^5 - 27x^7 + \dots, \quad |3x^2| < 1$$

A. $x - 3x^3 + 9x^5$

B. $2x + 9x^3 + 81x^5$

C. $\frac{1}{3}x - \frac{1}{9}x^2 - \frac{1}{27}x^3$

D. $\frac{1}{3}x - \frac{1}{9}x^3 + \frac{1}{27}x^5$

E. $1 - 3x^2 + 9x^4$

4. Find the first three terms of the power series for

$$f(x) = \frac{1}{\sqrt{1+2x}}$$

$$f(x) = (1+2x)^{-1/2} \quad \text{use binomial series}$$

$$= 1 + \left(-\frac{1}{2}\right)(2x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}(2x)^2 + \dots$$

$$= 1 - x + \frac{3}{2}x^2 + \dots$$

A. $1 - \frac{x}{2} + \frac{3x^2}{8}$

B. $1 + x - \frac{x^2}{2}$

C. $1 + \frac{x}{2} - \frac{x^2}{8}$

D. $1 - x + \frac{3x^2}{2}$

E. $1 - x - 3x^2$

5. Find the Taylor polynomial $p_3(x)$ about $a = 1$ for the function $f(x) = x^4 - 3x^2 + x + 1$.

$$f(x) = x^4 - 3x^2 + x + 1$$

$$f(1) = 0$$

$$f'(x) = 4x^3 - 6x + 1$$

$$f'(1) = -1$$

$$f''(x) = 12x^2 - 6$$

$$f''(1) = 6$$

$$f'''(x) = 24x$$

$$f'''(1) = 24$$

A. $1 + (x-1) - 2(x-1)^2$

B. $-(x-1) + 6(x-1) + 24(x-1)^3$

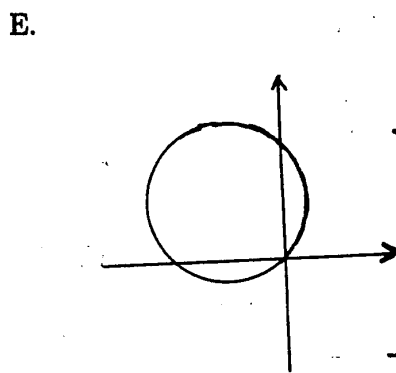
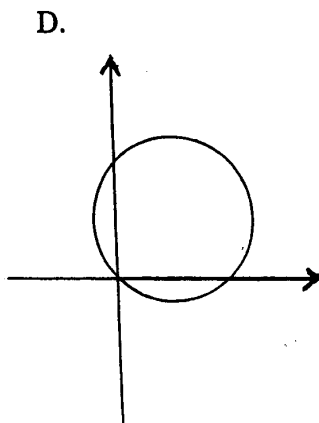
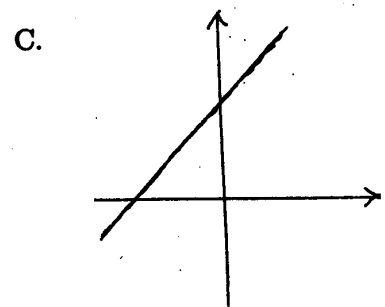
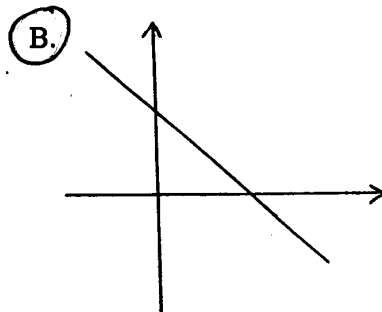
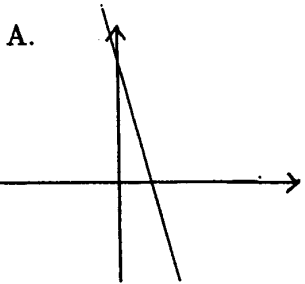
C. $1 + (x-1) - 4(x-1)^2$

D. $-(x-1) + 3(x-1)^2 + 4(x-1)^3$

E. $-(x-1) + 3(x-1)^2 + 8(x-1)^3$

$$\begin{aligned} p_3(x) &= 0 - 1(x-1) + \frac{6}{2!}(x-1)^2 + \frac{24}{3!}(x-1)^3 \\ &= -(x-1) + 3(x-1)^2 + 4(x-1)^3 \end{aligned}$$

6. Which of the curves below is described by the parameter equations $x = 1 + 2t$, $y = 2 - 2t$.



$$x = 1 + 2t \rightarrow t = \frac{x-1}{2}$$

$$\rightarrow y = 2 - 2\left(\frac{x-1}{2}\right)$$

$$= 2 - (x-1)$$

$$= 2 - x + 1$$

$$= -x + 3$$

\rightarrow choice B is best.

7. An object is moving in the plane. If its position at time t is $(1 - \cos^2 t, \sin 2t)$, then its velocity when $t = \frac{\pi}{4}$ is

$$r(t) = (1 - \cos^2 t) i + (\sin 2t) j \quad \text{A. } \frac{\sqrt{5}}{2}$$

$$v(t) = r'(t) = (-2 \cos t (-\sin t)) i + (2 \cos 2t) j \quad \text{B. } \sqrt{\frac{3}{2}}$$

$$v\left(\frac{\pi}{4}\right) = \left(2 \cos \frac{\pi}{4} \sin \frac{\pi}{4}\right) i + \left(2 \cos \frac{\pi}{2}\right) j \quad \text{C. } \frac{1}{2}$$

$$= \left(2 \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2}\right) i + (2 \cdot 0) j \quad \text{D. } \frac{\sqrt{2}}{2}$$

$$= i + 0j \quad \text{E. } 1$$

$$\|v\left(\frac{\pi}{4}\right)\| = 1$$

8. The length of the curve $x = \frac{t^3}{3} - t$, $y = t^2$, $1 \leq t \leq 2$, is

$$L = \int_1^2 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt \quad \text{A. } \frac{10}{3}$$

$$= \int_1^2 \sqrt{(t^2 - 1)^2 + (2t)^2} dt \quad \text{B. } \frac{14}{3}$$

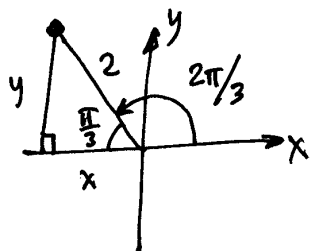
$$= \int_1^2 \sqrt{(t^4 - 2t^2 + 1) + 4t^2} dt \quad \text{C. } \frac{16}{5}$$

$$= \int_1^2 \sqrt{t^4 + 2t^2 + 1} dt \quad \text{D. } \frac{19}{5}$$

$$= \int_1^2 \sqrt{(t^2 + 1)^2} dt = \int_1^2 (t^2 + 1) dt \quad \text{E. } 4$$

$$= \left(\frac{1}{3} t^3 + t\right) \Big|_1^2 = \left(\frac{8}{3} + 2\right) - \left(\frac{1}{3} + 1\right) = \frac{14}{3} - \frac{4}{3} = \frac{10}{3}$$

9. If the polar coordinates of a point are $(2, \frac{2\pi}{3})$, then its Cartesian coordinates are



$$x = -1, y = \sqrt{3}$$

A. $(\sqrt{3}, 2\sqrt{3})$

B. $(-\sqrt{3}, 2)$

C. $(-1, \sqrt{3})$

D. $(\frac{1}{2}, \frac{3}{2})$

E. $(1, -\frac{\sqrt{3}}{2})$

(OR)

$$x = r \cos \theta = 2 \cos \frac{2\pi}{3} = 2 \left(-\frac{1}{2}\right) = -1$$

$$y = r \sin \theta = 2 \sin \frac{2\pi}{3} = 2 \left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$$

10. If the polar equation of a curve is $2r \cos \theta = \tan \theta$, what is its Cartesian equation?

A. $x^2 + y = 1$

B. $2x^2y^2 = 1$

C. $xy = y^2 + 1$

D. $2x^2y = 1$

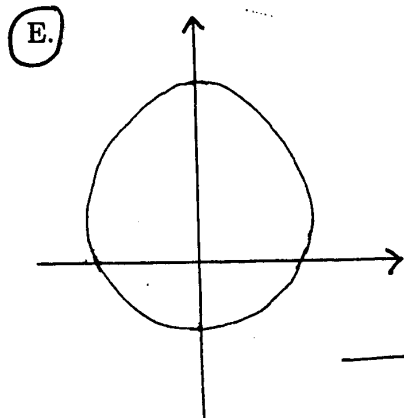
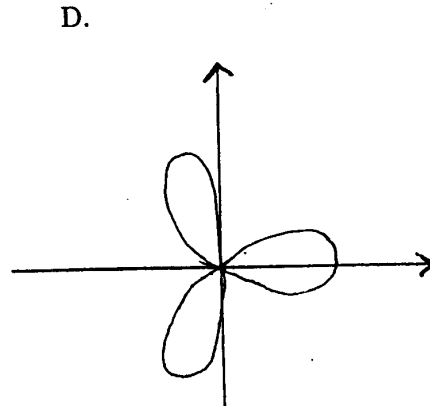
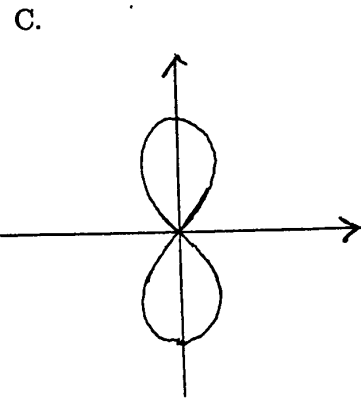
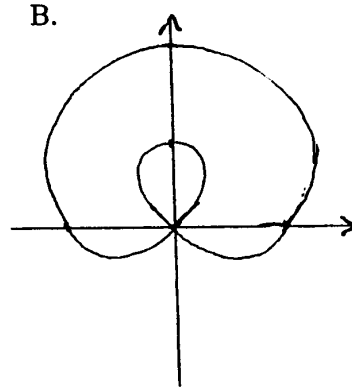
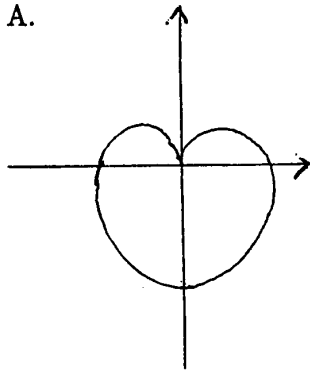
E. $y = 2x^2$

$$2(r \cos \theta) = \tan \theta$$

$$\rightarrow 2x = \frac{y}{x}$$

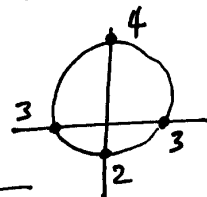
$$\rightarrow 2x^2 = y$$

11. The equation $r = 3 + \sin \theta$ describes the curve



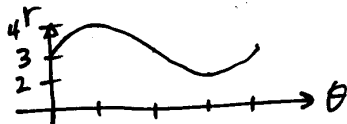
Variation of r with θ

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$3 + \sin \theta$	$3 \nearrow$	$4 \searrow$	$3 \searrow$	$2 \nearrow$	3



\rightarrow answer is E.

OR $r = 3 + \sin \theta$ on cartesian coord:



then wrap around the pole in cartesian coord.