MA 262

ALTERNATE FINAL EXAM   FALL 2000

NAME ______________________   INSTRUCTOR ______________________

1. Fill in your name and your instructor's name above and on the first page of the question sheets.
2. There are 25 questions, each worth 8 points.
3. NO CALCULATORS, BOOKS OR PAPERS ARE ALLOWED.. Use the back of the test pages for scrap paper.

1. If $y(x)$ is the solution of $\frac{dy}{dx} = \frac{x^2+1}{y^3}$ and $y(0) = 2$ then $y(3) =$

A. ± 2

B. $\pm 4\sqrt{2}$

C. $\pm 2\sqrt{2}$

D. 0

E. ± 4

2. The change of variable $u = y/x$ in the equation $y' = \tan\left(\frac{x^2+y^2}{x^2}\right)$ leads to the equation:

A. $xu' = \tan(u^2+1)$

B. $xu' = \tan(u^2+1) - x$

C. $xu' = \tan(u^2+1) + x$

D. $xu' = \tan(u^2+1) + u$

E. $xu' = \tan(u^2+1) - u$

3. The general solution of $xy' - y = x^2e^x$ is

A. $y = xe^x + cx$

B. $y = x^2e^x - xe^x + cx$

C. $y = xe^x - cx^2$

D. $y = x^2e^x + xe^x + cx$

E. None of the above

4. The solution of $(3x^2 + y)dx + (x + 2y)dy = 0$ passing through the point $(1, 1)$ is

A. $x^2 + xy + y^2 = 3$

B. $x^2 + xy + y^3 = 3$

C. $x^2 + x + y^2 = 3$

D. $x^3 + xy + y^2 = 3$

E. $x^3 + x^2y + y^3 = 3$

5. A tank contains 100 gallon of salt water which contains 10 lbs of salt. A salt solution of 2 lbs of salt per gallon enters the tank at a rate of 3 gallons per minute while a flow of fresh water runs into the tank at a rate of 5 gallons per minute. The well-stirred solution runs out of the tank at a rate of 7 gallons per minute. Let $A(t)$ be the amount of salt (in lbs) at the time t (in minutes). The initial value problem for $A(t)$ is

A. $\dfrac{dA}{dt} = 16 - \dfrac{7A}{t + 100} \qquad A(0) = 10$

B. $\dfrac{dA}{dt} = 6 - \dfrac{7A}{t + 100} \qquad A(0) = 10$

C. $\dfrac{dA}{dt} = 6 - \dfrac{8A}{t + 100} \qquad A(0) = 1000$

D. $\dfrac{dA}{dt} = 6 - \dfrac{8A}{t + 100} \qquad A(0) = 10$

E. $\dfrac{dA}{dt} = 16 - \dfrac{7A}{t + 100} \qquad A(0) = 1000$

6. The following system of linear equations has an infinite number of solutions:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= 0, \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= 1, \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= 0, \end{aligned}$$

In this case the matrix $A = (a_{ij}) \in R^{3\times 3}$ satisfies the condition

A. A is defective

B. A is nonsingular

C. The homogeneous systems $A\mathbf{x} = 0$ has exactly one solution

D. The homogeneous system $A\mathbf{x} = 0$ has more than one solution

E. None of the above

7. Let A be a 4×4 matrix. If $\mathbf{p} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix}$ and $\mathbf{q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$ are solutions of the system of linear equations $A\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 3 \end{bmatrix}$, then which of the following is also a solution?

A. $\mathbf{p} - \mathbf{q}$

B. $\mathbf{q} - 2\mathbf{p}$

C. $2\mathbf{p} - \mathbf{q}$

D. All of the above

E. None of the above

8. The entry b_{32} of the matrix $A^{-1} = (b_{ij})$ inverse to $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 4 & 1 \\ 2 & -4 & 0 \end{bmatrix}$ is equal to

A. $-\dfrac{1}{2}$

B. $\dfrac{3}{2}$

C. $\dfrac{2}{3}$

D. 2

E. 4

9. Let A_{ij} be the cofactor of the element a_{ij} in the matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ with $\det(A) = 5$. The value of the expression $a_{11}A_{11} + a_{12}A_{12} + a_{21}A_{21} + a_{22}A_{22}$ then is equal to

A. 0

B. 5

C. 10

D. 15

E. undetermined by the information given above

10. Let $A = \begin{bmatrix} 1 & k \\ k & 3 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. The values of k for which the system $A\mathbf{x} = b$ has a unique solution are

A. $k \neq \sqrt{6}$

B. $k \neq \{\sqrt{3}, -\sqrt{3}\}$

C. $k = \{\sqrt{2}, -\sqrt{2}\}$

D. All real numbers

E. None of the above

11. Consider the vectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 3 \\ 1 \\ 7 \\ 3 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 5 \\ -3 \\ 9 \\ 1 \end{bmatrix}$, $\mathbf{v}_4 = \begin{bmatrix} -2 \\ 4 \\ 2 \\ 8 \end{bmatrix}$. The dimension of the space $\text{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is then equal to

A. 1

B. 2

C. 3

D. 4

E. 5.

12. The subset $S = \{A \in M_3(\mathbb{R}); A = -A^T\}$ of the space of 3×3 matrices with real elements is a vector subspace such that

A. $\dim S = 2$ and a basis for S is $\left\{ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \right\}$

B. $\dim S = 2$ and a basis for S is $\left\{ \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \right\}$

C. $\dim S = 3$ and a basis for S is $\left\{ \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -2 & 0 \\ 2 & 0 & 2 \\ 0 & -2 & 0 \end{bmatrix} \right\}$

D. $\dim S = 3$ and a basis for S is $\left\{ \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix} \right\}$

E. None of the above

13. Which of the following sets S are subspaces of the given vector space V?

(i) $S = \{(x, y) \mid x \geq 0\}$ in $V = \mathbb{R}^2$

(ii) $S = \{f \in V \mid f(1) + f(-1) = 0\}$ in $V = P_3 = \{\text{real polynomials of degree} < 3\}$

(iii) $S = \{A \in V \mid \det A = 1\}$ in $V = M_2(\mathbb{R}) = \{2 \times 2 \text{ matrices with real entries}\}$

(iv) $S = \{(x, y, z) \mid 5x + 2y = z\}$ in $V = \mathbb{R}^3$

(v) $S = \{f \in V \mid f'' + f - 1 = 0\}$ in $V = P = \{\text{polynomials in } x\}$

A. (ii), (iv), (v)

B. (i), (ii), (iv)

C. (ii) and (iv)

D. (iii) and (iv)

E. (iv) and (v)

14. Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be defined by $T(\mathbf{x}) = A\mathbf{x}$ where $A = \begin{bmatrix} 1 & -1 & 1 & 3 \\ 0 & 1 & 1 & 2 \\ 2 & -1 & 3 & 8 \end{bmatrix}$. Then a basis for Ker$(T)$ is given by the vectors

A. $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 \\ -1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -3 \\ 8 \end{bmatrix}$

C. $\begin{bmatrix} 1 \\ 0 \\ 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \end{bmatrix},$

D. $\begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \\ 0 \\ 1 \end{bmatrix},$

E. $\begin{bmatrix} -2 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ -2 \\ 0 \\ 1 \end{bmatrix}$

15. Which of the following matrices are nondefective:

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}?$$

A. A, B

B. A, C

C. B only

D. C only

E. A only

16. One eigenvalue of $\begin{bmatrix} 1 & 2 & 2 \\ -1 & 4 & 1 \\ 0 & 0 & 3 \end{bmatrix}$ is $\lambda = 3$. A basis for the corresponding eigenspace is

A. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \right\}$

D. $\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$

E. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

17. If $y(x)$ is a solution of $y'' - 2y' + y = 0$ satisfying $y(0) = 1$ and $y'(0) = -1$, then $y(\frac{1}{2}) =$

A. 0

B. $-e^{\frac{1}{2}}$

C. $e^{\frac{1}{2}}$

D. $2e^{\frac{1}{2}}$

E. $-3e^{\frac{1}{2}}$

18. To find a particular solution of the differential equation

$$(D-1)^2(D-2)(D^2+1)y = e^x + \cos x - 2\sin x$$

one can use the following trial solution

A. $A_0 + A_1 \cos x + A_2 \sin x$

B. $A_0 e^x + x(A_1 \cos x + A_2 \sin x)$

C. $A_0 x e^x + x(A_1 \cos x + A_2 \sin x)$

D. $A_0 x^2 e^x + x^2(A_1 \cos x + A_2 \sin x)$

E. $A_0 x^2 e^x + x(A_1 \cos x + A_2 \sin x)$

19. The general solution of the differential equation $y'' + 4y = -8\dfrac{1}{\sin x}$ is

A. $2(\cos 2x)\ln|\sin 2x| + 4x\sin 2x$

B. $2(\sin 2x)\ln|\sin 2x| + 4x\cos 2x$.

C. $2\ln|\cos 2x| + 4x\cos 2x$

D. $2x\cos 2x + 4\sin 2x$

E. None of the above

20. In order to use the method of reduction of order to determine the general solution of $x^2y'' + 3xy' + y = 4\ln x$ for $x > 0$ one looks for a solution of the form

A. $y = xu(x)$

B. $y = x^{-1}u(x)$

C. $y = e^{x}u(x)$

D. $y = e^{-x}u(x)$

E. $y = xe^{x}u(x)$

21. Consider a damped spring-mass system whose motion is governed by

$$x'' + 2x' + 5x = 17\sin 2t, \quad x(0) = -2, \quad x'(0) = 0.$$

The solution of this initial value problem is

A. $x = 2e^{-t}\cos 2t - 4\cos 2t + \sin 2t$

B. $x = 2e^{-t}\sin 2t - 2\cos 2t + \sin 2t$

C. $x = e^{-t}\cos 2t - 3\cos 2t + \sin 2t$

D. $x = e^{-t}\sin 2t - 2\cos 2t + \sin 2t$

E. None of the above

22. One solution of $\mathbf{x}' = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \mathbf{x} + \mathbf{b}(t)$ is $\mathbf{x}(t) = \begin{bmatrix} t^{-2} \\ \ln t \end{bmatrix}$. The general solution is

A. $c_1 \begin{bmatrix} e^t \\ -e^t \end{bmatrix} + c_2 \begin{bmatrix} e^{3t} \\ e^{3t} \end{bmatrix} + \begin{bmatrix} t^{-2} \\ \ln t \end{bmatrix}$

B. $c_1 \begin{bmatrix} e^t \\ e^t \end{bmatrix} + c_2 \begin{bmatrix} e^{3t} \\ -e^{3t} \end{bmatrix} + c_3 \begin{bmatrix} t^{-2} \\ \ln t \end{bmatrix}$

C. $c_1 \begin{bmatrix} e^t \\ -e^t \end{bmatrix} + c_2 \begin{bmatrix} e^{3t} \\ e^{3t} \end{bmatrix} + c_3 \begin{bmatrix} t^{-2} \\ \ln t \end{bmatrix}$

D. $c_1 \begin{bmatrix} t^{-2} \\ \ln t \end{bmatrix}$

E. $c_1 \begin{bmatrix} e^t \\ e^t \end{bmatrix} + c_2 \begin{bmatrix} e^{3t} \\ -e^{3t} \end{bmatrix} + \begin{bmatrix} t^{-2} \\ \ln t \end{bmatrix}$

23. If $\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ is the solution of $\mathbf{x}' = \begin{bmatrix} 2 & 1 \\ -3 & -2 \end{bmatrix} \mathbf{x}$, $\mathbf{x}(0) = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$, then $x_1(1) =$

A. $-3e^{-1}$

B. e

C. e^{-1}

D. $-e$

E. $e + e^{-1}$

24. The general solution of $\mathbf{x}' = \begin{bmatrix} 3 & 5 \\ -1 & -1 \end{bmatrix} \mathbf{x}$ has the form

A. $c_1 e^t \begin{bmatrix} 2\cos t - \sin t \\ -\cos t \end{bmatrix} + c_2 e^t \begin{bmatrix} \cos t + 2\sin t \\ -\sin t \end{bmatrix}$

B. $c_1 e^t \begin{bmatrix} \cos t \\ 2\cos t - \sin t \end{bmatrix} + c_2 e^t \begin{bmatrix} -\sin t \\ \cos t - 2\sin t \end{bmatrix}$

C. $c_1 e^t \begin{bmatrix} 2\cos t + \sin t \\ -\cos t \end{bmatrix} + c_2 e^t \begin{bmatrix} \cos t + 2\sin t \\ \sin t \end{bmatrix}$

D. $c_1 e^t \begin{bmatrix} \cos t - 2\sin t \\ \sin t \end{bmatrix} + c_2 e^t \begin{bmatrix} 2\cos t + \sin t \\ -\cos t \end{bmatrix}$

E. None of the above.

25. The first order system of linear differential equations $\mathbf{x}' = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix} \mathbf{x}$ is equivalent to the following equation or system:

A. $y''' + y'' + 2y' + 3y = 0$

B. $y''' - y'' - 2y' - 3y = 0$

C. $y''' - 3y'' - 2y' - y = 0$

D. $\begin{cases} y'' + y + z = 0 \\ z' + z + 2y' + 3y'' = 0 \end{cases}$

E. $\begin{cases} y'' - y - z = 0 \\ z' - 3z - 2y' - y = 0 \end{cases}$