Solutions to Selected Problems

MA 262

\# 13 P 299. The solution space S of the equation $x_1 - 2x_2 - x_3 = 0$ is the set of vectors (x_1, x_2, x_3), where $x_2 = r$, $x_3 = s$, r, s arbitrary real numbers and $x_1 = 2r + s$. Thus

$$S = \{\vec{x} \in \mathbb{R}^3 : \vec{x} = (2r+s, r, s), \ r, s \text{ real}\}.$$

Now:

$$(2r+s, r, s) = r(2, 1, 0) + s(1, 0, 1).$$

Thus, $(2, 1, 0)$ and $(1, 0, 1)$ span S.

\# 14 P 299.

The null space of A is the set of solutions of $A\vec{x} = \vec{0}$. Solving this system of equations by Gauss elimination gives:

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -4 \\ 0 & -4 & -8 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

or:

$$x_1 + 2x_2 + 3x_3 = 0$$
$$x_2 + 2x_3 = 0$$

or: $x_2 = -2x_3$

$x_1 = -2x_2 - 3x_3 = 4x_3 - 3x_3 = x_3$

$\therefore$ null space$(A) = \{\vec{x} \text{ in } \mathbb{R}^3 : x = (r, -2r, r) \ r \text{ real}\}$

$= \{\vec{x} \text{ in } \mathbb{R}^3 \ \vec{x} = r(1, -2, 1)\}$.

$\therefore$ $(1, -2, 1)$ spans null space(A).

P 311 # 16.

We must determine whether $c_1 A_1 + c_2 A_2 = 0$ can only occur if $c_1 = c_2 = 0$

$$c_1 A_1 + c_2 A_2 = c_1 \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} + c_2 \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 2c_1 - c_2 & -c_1 + 2c_2 \\ 3c_1 + c_2 & 4c_1 + 3c_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

n:

$$\begin{aligned} 2c_1 - c_2 &= 0 \\ -c_1 + 2c_2 &= 0 \\ 3c_1 + c_2 &= 0 \\ 4c_1 + 3c_2 &= 0 \end{aligned}$$

Now:

$$\begin{aligned} 2c_1 - c_2 &= 0 \\ -c + 2c_2 &= 0 \end{aligned} \Rightarrow \begin{aligned} 2c_1 - c_2 &= 0 \\ -2c_1 + 4c_2 &= 0 \end{aligned} \Rightarrow 3c_2 = 0 \Rightarrow c_2 = 0$$

Sub. into $2c_1 - c_2 = 0 \Rightarrow c_1 = 0$

$\therefore$ A_1 and A_2 are lin indep.