Homework Probs MA 262.

P 322 # 7

P_4 = {all polynomials of degree < 4} = {all polynomials of the form $a_3x^3 + a_2x^2 + a_1x + a_0$.

Thus P_4 is spanned by

$1, x\ x^2\ x^3$

We assert that these functions are linearly indep't. To prove this assertion we calculate their Wronskian

$$\begin{vmatrix} 1 & x & x^2 & x^3 \\ 0 & 1 & 2x & 3x^2 \\ 0 & 0 & 2 & 6x \\ 0 & 0 & 0 & 6 \end{vmatrix} = 12 \neq 0$$

$\therefore$ $1, x, x^2, x^3$ are linearly independent. Since they are lin indep and span, they are a ~~p~~ basis for P_4. Thus P_4 has dimension four.

P 322 #10

Find dimension of null space A if

$$A = \begin{bmatrix} 1 & -1 & 2 & 3 \\ 2 & -1 & 3 & 4 \\ 1 & 0 & 1 & 1 \\ 3 & -1 & 4 & 5 \end{bmatrix}$$

null space A = set of solutions of $A\vec{x} = \vec{0}$

We solve this system by Gauss elimination

$$\begin{bmatrix} 1 & -1 & 2 & 3 \\ 2 & -1 & 3 & 4 \\ 1 & 0 & 1 & 1 \\ 3 & -1 & 4 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 2 & 3 \\ 0 & 1 & -1 & -2 \\ 0 & 1 & -1 & -2 \\ 0 & 2 & -2 & -4 \end{bmatrix}$$

1) Mult row 1 by −2 + add to row 2
2) " 1 by −1 + " row 3
3) Mult row 1 by −3 + add to row 4

$$\sim_{\text{Mult row 4 by } \frac{1}{2}} \begin{bmatrix} 1 & -1 & 2 & 3 \\ 0 & 1 & -1 & -2 \\ 0 & 1 & -1 & -2 \\ 0 & 1 & -1 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 2 & 3 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Thus system $A\vec{x} = 0$ has same soln's as

$$x_2 - x_3 - 2x_4 = 0$$
$$x_1 - x_2 + 2x_3 + 3x_4 = 0$$

$\therefore \quad x_2 = (x_3 + 2x_4)$ & sub this into 2nd eqn gives

$$x_1 = (x_3 + 2x_4) - 2x_3 - 3x_4 = -x_3 - x_4$$

$\therefore$ Letting $x_4 = s \quad x_3 = r$ ~~the~~ null space A is

null space $A = \{(x_1, x_2, x_3, x_4) : (-r-s,\ r+2s,\ r,\ s)\}$

Thus any vector in null space A is given

$$(-r-s,\ r+2s,\ r,\ s) = r(-1, 1, 1, 0)$$
$$+ s(-1, 2, 0, 1)$$

Thus $\vec{v}_1$ $(-1, 1, 1, 0)$ and $\vec{v}_2 = (-1, 2, 0, 1)$ span null space A

They are linearly independent since

$$c_1(-1, 1, 1, 0) + c_2(-1, 2, 0, 1) = (0, 0, 0, 0)$$

$$\Rightarrow c_1 = c_2 = 0$$

$\therefore$ ~~so the~~ null space A has dimension 2 (No of vectors in ~~the~~ basis)