MA 262

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Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ Then $cA = \begin{bmatrix} ca_{11} & ca_{12} \\ ca_{21} & ca_{22} \end{bmatrix}$

$\det(A) = a_{11}a_{22} - a_{21}a_{12}$

$\det(cA) = c^2(a_{11}a_{22} - a_{21}a_{12}) = c^2(\det A)$

Hence $T: A \to \det(A)$ is non linear

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$1, x, x^2$ constitute a basis for P_3

$\therefore$ If we know $T(1)$, $T(x)$, $T(x^2)$ we know

$T(ax^2+bx+c)$ since

$$T(ax^2+bx+c) = aT(x^2) + bT(x) + cT(1) \quad (*)$$

But we are given

$$x^2+x-3 = T(x^2-1) = T(x^2) - T(1)$$

linearity of T

$$4x = T(2x) = 2T(x)$$

$$2(x+3) = T(3x+2) = 3T(x) + 2T(1)$$

Thus

$$\left.\begin{aligned} T(x^2) \quad - T(1) &= x^2+x-3 \\ T(x) \quad &= 2x \\ 3T(x) + 2T(1) &= 2(x+3) \end{aligned}\right\}$$

Subs second eqn into 3rd gives

$$3(2x) + 2T(1) = 2x + 6$$

$$\therefore T(1) = -2x + 3$$

Subs into first equation gives:

$$T(x^2) = (-2x+3) + x^2+x-3 = x^2-x$$

Substituting values of $T(1)$, $T(x)$, $T(x^2)$ into $(*)$, and equating coefficients of like powers gives the result