MA 262, H.W. Problems

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To find eigenvalues of $\begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix}$, we must find the zeros of

$$\det \begin{bmatrix} 3-\lambda & -2 \\ 4 & -1-\lambda \end{bmatrix} = (\lambda+1)(\lambda-3)+8 = \lambda^2-2\lambda+5$$

The roots of this polynomial are (using quadratic formula)

$$\lambda = \frac{2 \pm \sqrt{4-20}}{2} = 1 \pm 2i$$

Eigenvector corresponding to $\lambda = 1+2i$ is solution of

$$\begin{bmatrix} 3-(1+2i) & -2 \\ 4 & -1-(1+2i) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Solving By Gauss elimination

$$\begin{bmatrix} 2-2i & -2 \\ 4 & -2-2i \end{bmatrix} \sim \begin{bmatrix} 1-i & -1 \\ 2 & -(1+i) \end{bmatrix} \underset{\uparrow}{\sim} \begin{bmatrix} 2 & -(1+i) \\ 2 & -(1+i) \end{bmatrix}$$

mult row 1 by $(1+i)$

$$\sim \begin{bmatrix} 2 & -(1+i) \\ 0 & 0 \end{bmatrix}$$

$\therefore$ Eigenvectors corresponding to $\lambda + 2i$ have form

$$x_2 = r \quad x_1 = \frac{(1+i)}{2} r \quad \text{or} \quad r\left(\frac{1+i}{2}, 1\right) \quad r \neq 0$$

Since complex eigenvalues and corresponding eigenvectors occur in conjugate pairs, the eigenvectors corresponding to $\lambda = 1-2i$ have the form

$$r\left(\frac{1-i}{2}, 1\right) \quad r \neq 0$$

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$$A(3\vec{v}_1 - \vec{v}_2) = 3A\vec{v}_1 - A\vec{v}_2 = 3(\lambda_1\vec{v}_1) - \lambda_2\vec{v}_2$$

Properties of matrix mult.

$\vec{v}_1$ is eigenvector corresp to eigenvalue λ_1
$\vec{v}_2$ eigenvector corresp to eigenvalue λ_2

$$= 3\lambda_1\vec{v}_1 - \lambda_2\vec{v}_2$$

$$= 6(1,-1) + 3(2,1)$$

given values of $\lambda_1, \lambda_2, \vec{v}_1\ \vec{v}_2$