MA 262

P 401 # 19

To find eigenvectors corresponding to $\lambda = 2$ we solve the system

$$\begin{bmatrix} 1-(2) & -3 & 1 \\ -1 & -1-(2) & 1 \\ -1 & -3 & 3-(2) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Working with coefficient matrix we get

$$\begin{bmatrix} -1 & -3 & 1 \\ -1 & -3 & 1 \\ -1 & -3 & 1 \end{bmatrix} \sim \begin{bmatrix} -1 & -3 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ Eigenvectors satisfy

$$-V_1 - 3V_2 + V_3 = 0$$

or $\quad V_1 = -3V_2 + V_3$

$\therefore$ Eigenvectors have form

$$(-3r+s, r, s) = r(-3, 1, 0) + s(1, 0, 1)$$

Eigenspace corresponding to $\lambda = 2$ is spanned by the two lin indep vectors $(-3, 1, 0)$ $(1, 0, 1)$. These two vectors and the eigenvector corresponding to $\lambda = 1$ will be linearly independent. Therefore the matrix is non-defective