MA 262

Problem 22 P 442

$$y''' - y'' + y' - y' = 9e^{-x} \quad (*)$$

Auxiliary equation is

$$r^3 - r^2 + r - 1 = 0$$

$r=1$ is a root. Therefore $r^3-r^2+r-1 = (r-1)q(r)$

To find $q(r)$ use long division

$$\begin{array}{r|l} & r^2+1 \\ \hline r-1 & r^3 + r^2 + r - 1 \\ & r^3 + r^2 \\ \hline & r-1 \end{array}$$

$\therefore q(r) = (r^2+1)$

Thus DE can be written as

$P(D)y = 9e^{-x}$ where $P(D) = (D-1)(D^2+1)$

Annihilator of $9e^{-x}$ is $Q(D) = (D+1)$

$\therefore$ y_p is among the solutions of

$$Q(D)P(D)y = (D+1)(D-1)(D^2+1)y = 0$$

$$\therefore y_p = c_1 e^{-x} + c_2 e^{x} + c_3 \cos x + c_4 \sin x$$

But $y_c = c_2 e^{x} + c_3 \cos x + c_4 \sin x$

is the general solution of the homogeneous equation,

so we may take $y_p = Ce^{-x}$.

Plug this into $(*)$ + solve for C, to complete the problem.