MA266 Practice Problems

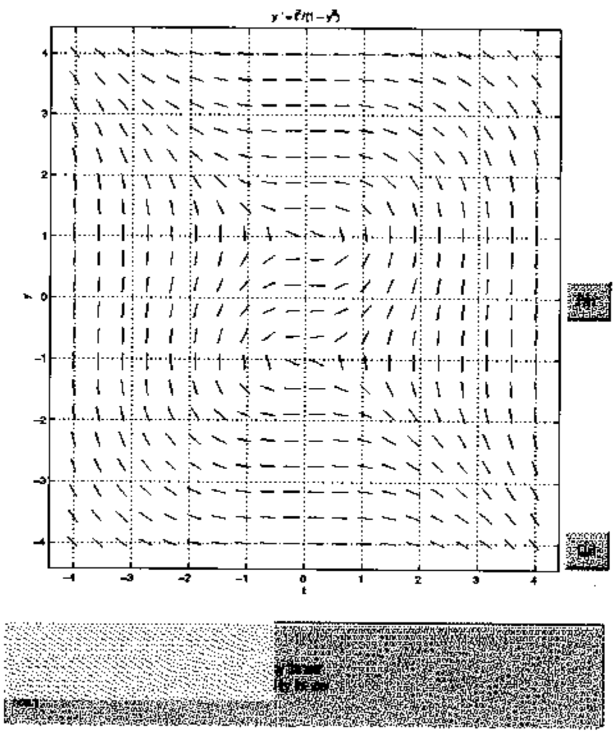
1. Which of $y_1(t) = t$, $y_2(t) = t^2$, $y_3(t) = t^3$ are solutions of $t^2y'' - 3ty' + 3y = 0$?

   A. only y_1 **B.** only y_2 **C.** y_1 and y_2 **D.** y_2 and y_3 **E.** y_1 and y_3

2. What is the largest open interval for which a unique solution of the initial value problem $ty' + \frac{1}{t+1}y = \frac{t-2}{t-3}$, $y(1) = 0$ is guaranteed ?

   A. $0 < t < 1$ **B.** $0 < t < 2$ **C.** $0 < t < 3$ **D.** $-1 < t < 3$ **E.** $-1 < t < 1$

3. Use the dfield plot below to estimate where the solution of $y' = \frac{t^2}{1-y^2}$, $y(0) = 0$ is defined:



   A. $-1.2 < t < 1.2$ **B.** $-4 < t < 4$ **C.** $-1 < t < 2$ **D.** $-2 < t < 2$ **E.** $-4 < t < \infty$

4. Consider the autonomous differential equation $\frac{dy}{dt} = -\frac{1}{10}(y-1)(y-4)^2$. Classify the stability of each equilibrium solution.

   A. $y = 1$ and $y = 4$ both unstable **B.** $y = 4$ stable; $y = 1$ unstable **C.** $y = 0$ and $y = 1$ stable; $y = 4$ unstable **D.** $y = 1$ stable; $y = 4$ unstable **E.** $y = 0$ stable; $y = 1$ and $y = 4$ unstable

5. Determine whether $x + 2y + (2x + y)\frac{dy}{dx} = 0$ is separable, homogeneous, linear and/or exact.

   A. LINEAR and SEP **B.** SEP and HOM **C.** HOM and EXACT **D.** LINEAR and HOM **E.** LINEAR, HOM and EXACT

6. An explicit solution of $y' = y^2 - 1$ is

A. $y = \dfrac{Ce^{2t}}{1 - Ce^{2t}}$ **B.** $y = \dfrac{1 + Ce^{2t}}{1 - Ce^{2t}}$ **C.** $y = \dfrac{1}{1 - Ce^{2t}}$ **D.** $y = \dfrac{1 + Ce^{2t}}{1 - e^{2t}}$ **E.** $\dfrac{y^3}{3} - y = C$

7. If $y' = y^3$ and $y(0) = 1$, then $y(-1) =$

A. $5^{-\frac{1}{4}}$ **B.** $\frac{1}{\sqrt{3}}$ **C.** $\sqrt{3}$ **D.** 1 **E.** Does not exist

8. The general solution of $y' + (1 + \frac{1}{t})\, y = \frac{1}{t}$ is

A. $y = t + C \ln t$ **B.** $y = Ct + \ln t$ **C.** **D.** $y = \dfrac{1}{t} + \dfrac{C}{t} e^{-t}$ **E.** $y = \dfrac{1}{t} + C$

9. An implicit solution of $y^2 + 1 + (2xy + 1)\dfrac{dy}{dx} = 0$ is

A. $2(xy^2 + y) = C$ **B.** $xy^2 + y = C$ **C.** $xy^2 + x + y = C$ **D.** $\frac{y^3}{3} + y + x^2 y + x = C$ **E.** $y = xy^2 + C$

10. If y' is proportional to y, $y(0) = 2$ and $y(1) = 8$. For what value of t does $y(t) = 20$?

A. $\ln 6$ **B.** $\ln 4$ **C.** $\dfrac{\ln 8}{\ln 2}$ **D.** $\ln \frac{5}{2}$ **E.** $\dfrac{\ln 10}{\ln 4}$

11. The general solution of $y'' - 4y' + 4y = 0$ is

A. $y = C_1 e^{2t} + C_2 t e^{2t}$ **B.** $y = C_1 e^{2t} + C_2 e^{2t}$ **C.** $y = C_1 e^{2t} + C_2 e^{-2t}$ **D.** $y = C_1 e^{-2t} + C_2 t e^{-2t}$ **E.** $y = C_1 t + c_2 t^2$

12. The general solution of $y''' + 4y'' + 5y' = 0$ is

A. $y = C_1 e^{-2t} \cos t + C_2 e^{-2t} \sin t$ **B.** $y = C_1 + C_2 e^{-2t} \cos t + C_3 e^{-2t} \sin t$ **C.** $y = C_1 + C_2 e^{t} \cos 2t + C_3 e^{t} \sin 2t$ **D.** $y = C_1 + C_2 \cos t + C_3 \sin t$ **E.** $y = C_1 + C_2 e^{2t} \cos t + C_3 e^{2t} \sin t$

13. A particular solution, y_p, of $y'' - 4y' + 3y = 2t + e^t$ is

A. $\frac{2}{3}t + \frac{8}{9} - \frac{1}{2}te^t$ **B.** $\frac{2}{3}t + \frac{1}{2} - \frac{1}{2}te^t$ **C.** $\frac{1}{3}t + \frac{1}{2} - \frac{1}{2}te^t$ **D.** $\frac{1}{3}t + \frac{1}{2} - \frac{1}{2}e^t$ **E.** $t^2 + e^t$

14. If $y'' + 5y' + 6y = 24e^t$, $y(0) = 0$, $y'(0) = 0$, then $y(1) =$

A. $-e^{-2} + 6e^{-3} + e$ **B.** $-8e^{-2} + 6e^{-3} + e$ **C.** $8e^{-2} + e^{-3} + e$ **D.** $-8e^{-2} + 6e^{-3} + 2e$ **E.** 0

15. The Wronskian $W(y_1, y_2)$ of $y_1(t) = t^2$ and $y_2(t) = t^{-2}$ is

A. $-2t - \frac{2}{t}$ **B.** $\frac{2}{t}$ **C.** $-\frac{4}{t}$ **D.** 1 **E.** 0

16. An object weighing 8 pounds attached to a spring will stretch it $\frac{1}{2}$ ft beyond its natural length. There is a damping force with a damping constant $c = 6$ lbs-sec/ft and there is no external force. If at $t = 0$ the object is pulled 2 feet below equilibrium and then released, the initial value problem describing the vertical displacement $x(t)$ becomes :

A. $\begin{cases} 8x'' + 6x' + 16x = 0 \\ x(0) = 2 \\ x'(0) = 0 \end{cases}$ **B.** $\begin{cases} 8x'' + 6x' + 16x = 0 \\ x(0) = -2 \\ x'(0) = 0 \end{cases}$ **C.** $\begin{cases} \frac{1}{4}x'' + 6x' + 16x = 0 \\ x(0) = 2 \\ x'(0) = 0 \end{cases}$

D. $\begin{cases} \frac{1}{4}x'' + 6x' + 8x = 0 \\ x(0) = 2 \\ x'(0) = 0 \end{cases}$ **E.** $\begin{cases} 256x'' + 6x' + 16x = 0 \\ x(0) = 2 \\ x'(0) = 0 \end{cases}$

17. A spring-mass system is governed by the initial value problem $x''+4x'+4x = 4\cos\omega t$, $x(0) = 0$, $x'(0) = -2$. For what value(s) of ω will resonance occur ?

A. 0 **B.** 2 **C.** 4 **D.** $2 < \omega < \infty$ **E.** no value of ω

18. A tank initially contains 40 ounces of salt mixed in 100 gallons of water. A solution containing 4 oz of salt per gallon is then pumped into the tank at a rate of 5 gal/min. The stirred mixture flows out of the tank at the same rate. How much salt is in the tank after 20 minutes ?

A. 20 **B.** 80 **C.** $40 + 20e$ **D.** $400 - 360e^{-1}$ **E.** $400 + 360e^2$

19. Write $2u'' + 3u' + ku = \cos 2t$ as a system of 1^{st} order equations.

A. $\begin{cases} x' = y \\ y' = \frac{1}{2}(-3y - kx + \cos 2t) \end{cases}$ **B.** $\begin{cases} x' = y \\ y' = \frac{1}{2}(-3x - ky + \cos 2t) \end{cases}$

C. $\begin{cases} x' = x \\ y' = \frac{1}{2}(-3y - kx + \cos 2t) \end{cases}$ **D.** $\begin{cases} x' = y \\ y' = 2y + kx + \cos 2t \end{cases}$

E. $\begin{cases} x' = 2y + 3x + \cos 2t \\ y' = x \end{cases}$

20. The solution of $X' = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix} X$, $X(0) = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ is

A. $2e^{3t}\begin{pmatrix} 1 \\ 2 \end{pmatrix} + e^{-t}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ **B.** $2e^{3t}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + e^{-t}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ **C.** $e^{3t}\begin{pmatrix} 2 \\ 1 \end{pmatrix} + e^{-t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

D. $3e^{3t}\begin{pmatrix} 1 \\ -2 \end{pmatrix} - e^{-t}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ **E.** $3e^{3t}\begin{pmatrix} 1 \\ 2 \end{pmatrix} + e^{-t}\begin{pmatrix} 0 \\ -4 \end{pmatrix}$

21. Solve $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, $X(0) = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$.

A. $X(t) = 2e^t\begin{pmatrix} \sin t \\ \cos t \end{pmatrix} - e^t\begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$ **B.** $X(t) = 2e^t\begin{pmatrix} \sin t \\ \cos t \end{pmatrix} + e^t\begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$

C. $X(t) = 2e^t\begin{pmatrix} \sin t \\ \cos t \end{pmatrix} - e^t\begin{pmatrix} \cos t \\ -\sin t \end{pmatrix}$ **D.** $X(t) = e^t\begin{pmatrix} \sin t \\ \cos t \end{pmatrix} - e^t\begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$

E. $X(t) = e^t\begin{pmatrix} -\sin t \\ \cos t \end{pmatrix} - e^t\begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$

22. Solve the initial value problem $\vec{\mathbf{x}}'(t) = A\vec{\mathbf{x}}(t)$, $\vec{\mathbf{x}}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ where $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$.

A. $e^t\begin{pmatrix} 1 \\ 1 \end{pmatrix} - 2te^t\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ **B.** $e^t\begin{pmatrix} 1 \\ 1 \end{pmatrix} + 2te^t\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ **C.** $e^t\begin{pmatrix} 1 \\ 1 \end{pmatrix} + te^t\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

D. $e^t\begin{pmatrix} 0 \\ 1 \end{pmatrix} + 2te^t\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ **E.** $e^t\begin{pmatrix} 1 \\ 1 \end{pmatrix} - 2te^t\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

23. Find a particular solution of $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

A. $X_p = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ **B.** $X_p = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ **C.** $X_p = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ **D.** $X_p = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$
E. $X_p = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

24. Find the general solution of $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} - \begin{pmatrix} 6e^{-t} \\ 1 \end{pmatrix}$.

A. $C_1e^t\begin{pmatrix} 0 \\ 1 \end{pmatrix} + C_2e^{2t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

B. $C_1e^t\begin{pmatrix} 0 \\ 1 \end{pmatrix} + C_2e^{2t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 6e^{-t} \\ 1 \end{pmatrix}$

C. $C_1e^t\begin{pmatrix} 0 \\ 1 \end{pmatrix} + C_2e^{2t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + e^{-t}\begin{pmatrix} 6 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

D. $C_1e^t\begin{pmatrix} 0 \\ 1 \end{pmatrix} + C_2e^{2t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + e^{-t}\begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

E. $C_1e^t\begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2e^{2t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + e^{-t}\begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

25. $\mathcal{L}\{e^t(1+\cos 2t)\} =$

A. $\dfrac{1}{s-1} + \dfrac{1}{(s-1)^2+4}$ **B.** $\dfrac{1}{s-1} + \dfrac{s-1}{s^2-2s+5}$ **C.** $\left(\dfrac{1}{s-1}\right)\left(\dfrac{1}{s} + \dfrac{s-1}{(s-1)^2+4}\right)$
D. $\left(\dfrac{1}{s-1}\right)\left(\dfrac{s-1}{s^2-2s+5}\right)$ **E.** $\dfrac{1}{s} + \dfrac{s-1}{(s-1)^2+4}$

26. Find the Laplace transform of $f(t) = \begin{cases} t, & 0 \le t < 1 \\ 0, & 1 \le t < \infty \end{cases}$.

A. $e^{-s}\left(\dfrac{1}{s} + \dfrac{1}{s-2}\right)$ **B.** $\dfrac{1}{s^2} + 2e^{-s}\left(\dfrac{1}{s} + \dfrac{1}{s^2}\right)$ **C.** $\dfrac{1}{s^2} - e^{-s}\dfrac{1}{s^2}$ **D.** $\dfrac{1}{s^2} - e^{-s}\left(\dfrac{1}{s} + \dfrac{1}{s^2}\right)$
E. $e^{-s}\left(\dfrac{1}{s} + \dfrac{1}{s^2}\right)$

27. Solve $y'' + 3y' + 2y = 4\mathcal{U}(t-1)$, $y(0) = 0$, $y'(0) = 1$

A. $\mathcal{U}(t-1)\left(2 - 4e^{-(t-1)} + 2e^{-2(t-1)}\right)$ **B.** $\mathcal{U}(t-1)\left(2 - 4e^{-(t-1)} + 2e^{-2(t-1)}\right) + e^{-t} - e^{-2t}$
C. $\mathcal{U}(t)\left(2 - 4e^{-(t-1)} + 2e^{-2(t-1)}\right) + e^{-t} - e^{-2t}$ **D.** $\left(2 - 4e^{-(t-1)} + 2e^{-2(t-1)}\right) + e^{-t} - e^{-2t}$
E. $e^{-t} - e^{-2t}$

28. Find the solution of the initial value problem $y'' + y = \delta(t-\pi)$, $y(0) = 0$, $y'(0) = 1$.

A. $y = \sin t + \mathcal{U}(t)\sin t$ **B.** $y = \sin t + \mathcal{U}(t-\pi)\sin \pi t$ **C.** $y = \sin t + \mathcal{U}(t-\pi)\sin(t-\pi)$
D. $y = \mathcal{U}(t-\pi)(\sin t + \sin(t-\pi))$ **E.** $y = \mathcal{U}(t-\pi)\sin t$

29. The inverse Laplace transform of $F(s) = \dfrac{se^{-s}}{s^2+2s+5}$ is

A. $\mathcal{U}(t-1)\,(e^{-t}\cos 2(t-1)) - \frac{1}{2}\,e^{-t}\sin 2(t-1)$ **B.** $(e^{-t+1}\cos 2(t-1)) - \frac{1}{2}\,e^{-t+1}\sin 2(t-1)$
C. $\mathcal{U}(t-1)\,(e^{t-1}\cos 2(t-1)) - \frac{1}{2}\,e^{t-1}\sin 2(t-1)$ **D.** $\mathcal{U}(t)\,(e^{-t}\cos 2t) - \frac{1}{2}\,e^{-t}\sin 2t$
E. $\mathcal{U}(t-1)\left(e^{-t+1}\cos 2(t-1) - \frac{1}{2}\,e^{-t+1}\sin 2(t-1)\right)$

30. $\mathcal{L}\left\{\int_0^t \sin 2(t-\nu)\,\cos(3\nu)\,d\nu\right\} =$

A. $\dfrac{2s}{(s^2+4)(s^2+9)}$ **B.** $\dfrac{2}{s^2+4}+\dfrac{s}{s^2+9}$ **C.** $\dfrac{1}{s^2+4}+\dfrac{s}{s^2+9}$ **D.** $\dfrac{2}{(s^2+4)(s^2+9)}$
E. $\dfrac{s}{(s^2+4)(s^2+9)}$

Answers

1. E
2. C
3. A
4. D
5. C
6. B
7. B
8. D
9. C
10. E

11. A
12. B
13. A
14. D
15. C
16. C
17. E
18. D
19. A
20. A

21. C
22. C
23. A
24. D
25. B
26. D
27. B
28. C
29. E
30. A

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\dfrac{1}{s}$
2.	e^{at}	$\dfrac{1}{s-a}$
3.	t^n	$\dfrac{n!}{s^{n+1}}$
4.	$t^n e^{at}$	$\dfrac{n!}{(s-a)^{n+1}}$
5.	$\cos kt$	$\dfrac{s}{s^2+k^2}$
6.	$\sin kt$	$\dfrac{k}{s^2+k^2}$
7.	$e^{at}\cos kt$	$\dfrac{s-a}{(s-a)^2+k^2}$
8.	$e^{at}\sin kt$	$\dfrac{k}{(s-a)^2+k^2}$
9.	$\sqrt{t}$	$\dfrac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$
10.	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
11.	$f'(t)$	$sF(s) - f(0)$
12.	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
13.	$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - sf^{(n-2)}(0) - f^{(n-1)}(0)$
14.	$e^{at}f(t)$	$F(s-a)$
15.	$\mathcal{U}(t-a)$	$\dfrac{e^{-as}}{s}$
16.	$f(t-a)\,\mathcal{U}(t-a)$	$e^{-as}F(s)$
17.	$f(t)\,\mathcal{U}(t-a)$	$e^{-as}\mathcal{L}\{f(t+a)\}$
18.	$\delta(t-t_0)$	$e^{-t_0 s}$
19.	$(f*g)(t) = \int_0^t f(t-\nu)\,g(\nu)\,d\nu$	$F(s)\,G(s)$
20.	$f(t-T) = f(t)$	$\dfrac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)\,dt$