# Worksheet # 1

## Euler (Tangent Line) Method

(1) Approximate the actual solution of $\begin{cases} y' = x + 2y \\ y(0) = 1 \end{cases}$ at $x = 0.6$ using the Euler Method with $h = 0.2$. Do this by hand and show all computations.

(2) Find the actual solution $\phi(x)$ of the initial value problem above and use the Euler Method (**eul**) with $h = 0.2$ to complete this table:

| $x_n$ | Euler Approximation $y_n$ | Actual Solution $\phi(x_n)$ |
|---|---|---|
| 0.0 | | |
| 0.2 | | |
| 0.4 | | |
| 0.6 | | |
| 0.8 | | |
| 1.0 | | |
| 1.2 | | |
| 1.4 | | |

(3) Consider the initial value problem $y' = -2y + e^{-x}, \quad y(0) = 1$ .

(a) Solve this initial value problem and find $y(1)$.

$y =$ ______

$y(1) =$ ______

(b) What is the smallest value of $n$ for which the Euler Method (**eul**) with $n$ steps ($h = \frac{1}{n}$) will give a value $y_n$ that approximates the actual solution at $x = 1$ within 0.05 ?

$n =$ ______

(c) Use **dfield5** to plot (on the same graph) the solutions of $y' = -2y + e^{-x}$ satisfying $y(0) = 0.95$, $y(0) = 1$ and $y(0) = 1.05$.

(Attach the graph at the end of this worksheet)

(4) Consider the initial value problem $y' = 2y - 3e^{-x}$, $y(0) = 1$ .

(a) Solve this initial value problem and find $y(1)$.

$y =$ [ ]

$y(1) =$ [ ]

(b) What is the smallest value of $n$ for which the Euler Method (**eul**) with $n$ steps ($h = \frac{1}{n}$) will give a value $y_n$ that approximates the actual solution at $x = 1$ within 0.05 ?

$n =$ [ ]

(c) Use **dfield5** to plot (on the same graph) the solutions of $y' = 2y - 3e^{-x}$ satisfying $y(0) = 0.95$, $y(0) = 1$ and $y(0) = 1.05$.

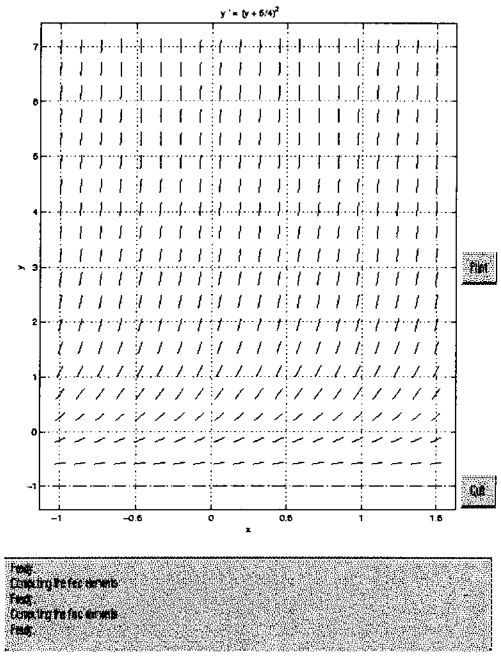
(Attach the graph at the end of this worksheet)

(5) Using the plots in (3)(c) and (4)(c) above, briefly explain why $n$ is larger in one case rather than the other to obtain the same degree of accuracy.

[ ]

(6) Give reasons why the Euler Method with $h = 0.1$ does not give a good approximation of the actual solution at $x = 1$ of these initial value problems:

(a) $y' = (y + \frac{5}{4})^2, \quad y(0) = 0 \qquad$ *Actual solution :* $\quad y = \dfrac{25x}{4(4 - 5x)}$




(b) $y' = \dfrac{25x}{32(1 - 2y)}, \quad y(0) = 0 \qquad$ *Actual solution :* $\quad y = \frac{1}{8}(4 - \sqrt{16 - 25x^2})$