WORKSHEET # 2

Euler, Improved Euler & Runge-Kutta Methods

(1) Consider the initial value problem $\begin{cases} y' = \sqrt{x^2 + 3y^2} \\ y(2) = 0.5 \end{cases}$ Using the the Euler, Improved Euler and Runge-Kutta Methods (**eul, rk2, rk4** respectively) with $h = 0.1$ to complete this table:

x_n	Euler y_n	Improved Euler y_n	Runge-Kutta y_n
2.0			
2.1			
2.2			
2.3			
2.4			
2.5			
2.6			

(2) Approximate the actual solution of the above initial value problem at $x = 2.6$ using the Euler Method (**eul**) with $h = 0.02$ and using the Runge-Kutta Method (**rk4**) with $h = 0.6$:

Euler Method approximation	
Runge-Kutta Method approximation	

(3) Approximation methods for differential equations can be used to estimate definite integrals:

(a) Show that $y(x) = \int_0^x e^{-t^2}\,dt$ satisfies the initial value problem $\frac{dy}{dx} = e^{-x^2}, \quad y(0) = 0$.

(b) Use the Runge-Kutta Method (**rk4**) with $h = 0.1$ to approximate $y(1.2)$:

$$y(1.2) = \int_0^{1.2} e^{-t^2}\,dt \approx \boxed{}$$

(c) Use the Euler Method (**eul**) with $h = 0.1$ to approximate $y(1.2)$. What is the relation of this to Riemann sums ?

(4) Choosing smaller and smaller step sizes h does not guarantee better and better approximations even for a simple initial value problem like $\begin{cases} \dfrac{dy}{dx} = x\,(y-1) \\ y(-10) = 0 \end{cases}$.

(a) Verify that $y(x) = 1 - e^{\frac{(x^2-100)}{2}}$ is a solution to this initial value problem.

(b) Approximate the actual solution at $x = 10$ (note that $y(10) = 0$) using the Runge-Kutta Method (**rk4**) with $h = 0.2, h = 0.1$ and $h = 0.05$ and fill in the table:

h	Runge-Kutta Approximation at $x = 10$	Actual Solution at $x = 10$
0.20		0.0000
0.10		0.0000
0.05		0.0000