From snider@math.purdue.edu Tue Jun 19 09:47:54 2001 Date: Tue, 19 Jun 2001 09:23:54 -0500 (E
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NAME ________________________________________

STUDENT ID ____________________________________

REC. INSTR. __________________ REC. TIME __________________

INSTRUCTIONS:

1. Verify that you have all the pages (there are 6 pages).
2. Fill in your name, your student ID number, and your recitation instructor's name and recitation time above. Write your name, your student ID number and division and section number of your recitation section on your answer sheet, and fill in the corresponding circles.
3. Mark the letter of your response for each question on the mark–sense answer sheet.
4. There are 12 problems worth 8 points each.
5. No books or notes or calculators may be used.

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$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \qquad |x| < \infty$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}, \qquad |x| < \infty$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}, \qquad |x| < \infty$$

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n, \qquad |x| < 1$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}, \qquad |x| < 1$$

Taylor series of $f(x)$ centered at $a$ :

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$$

1. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\frac{2^n}{5^{n+1}} =$

A. $0$

B. $\dfrac{2}{3}$

C. $\dfrac{2}{7}$

D. $\dfrac{2}{15}$

E. $\dfrac{2}{35}$

2. Let $S = \displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n^2+1}$. How many terms should we add to estimate $S$ with an error less than $10^{-2}$?

A. 7

B. 9

C. 11

D. 13

E. 15

3. Consider the following two series:

(I) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n+1}{n^3-n+1}$

(II) $\displaystyle\sum_{n=1}^{\infty}\frac{n+1}{n^3-n+1}$

A. Series (I) converges absolutely and series (II) converges

B. Series (I) converges absolutely and series (II) diverges

C. Series (I) converges conditionally and series (II) converges

D. Series (I) converges conditionally and series (II) diverges

E. Both series diverge

4. Consider the following two series:

(I) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\sqrt{n^2+n}}{n+2}$

(II) $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt{n^2+n}}{n+2}$

A. Series (I) converges absolutely and series (II) converges

B. Series (I) converges absolutely and series (II) diverges

C. Series (I) converges conditionally and series (II) converges

D. Series (I) converges conditionally and series (II) diverges

E. Both series diverge

5. Consider the following two series:

(I) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{3^n}$ (II) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{2^n}$

A. Both series converge

B. Both series diverge

C. Series (I) converges and series (II) diverges

D. Series (I) diverges and series (II) converges

E. None of the above

6. Consider the following two series:

(I) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)3^n}$ (II) $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)3^n}{n!}$

A. Both series converge

B. Both series diverge

C. Series (I) converges and series (II) diverges

D. Series (I) diverges and series (II) converges

E. None of the above

7. The radius of convergence of the series $\sum_{n=1}^{\infty} \frac{3^n x^n}{n^2 5^{n+1}}$ is

A. 5

B. $\frac{1}{3}$

C. $\frac{5}{3}$

D. $\frac{3}{5}$

E. 3

8. The interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n 2^n}$ is

A. $[-2, 2)$

B. $(-2, 2]$

C. $(-1, 3]$

D. $[-1, 3)$

E. $[-1, 3]$

9. $\int_0^x \tan^{-1}(2t)dt$ has power series representation given by

A. $\sum_{n=0}^{\infty} (-1)^n \frac{2}{(2n+1)(2n+2)} x^{2n+2}$

B. $\sum_{n=0}^{\infty} (-1)^n \frac{2^{2n+1}}{(2n+1)(2n+2)} x^{2n+2}$

C. $\sum_{n=0}^{\infty} (-1)^n \frac{2}{2n+1} x^{2n+1}$

D. $\sum_{n=0}^{\infty} (-1)^n \frac{2^{2n+2}}{(2n+1)(2n+2)} x^{2n+2}$

E. $\sum_{n=0}^{\infty} (-1)^n \frac{2}{(2n+2)!} x^{2n+2}$

10. Find the Taylor series for $\dfrac{1}{x^2}$ centered at $a = 1$.

A. $\sum_{n=0}^{\infty}(-1)^n n(x-1)^n$

B. $\sum_{n=0}^{\infty}(-1)^n \dfrac{(x-1)^n}{n+1}$

C. $\sum_{n=0}^{\infty}(-1)^n (n+1)(x-1)^n$

D. $\sum_{n=0}^{\infty}(-1)^n \dfrac{x^n}{n+1}$

E. $\sum_{n=0}^{\infty}(-1)^n (n+1)x^n$

11. The first three terms of the binomial series expansion of the function $\sqrt[4]{1+16x^2}$ are given by

A. $1+4x^2-24x^4$

B. $1-4x^2+24x^4$

C. $1+4x-24x^2$

D. $1-4x+24x^2$

E. $1+4x^2-32x^4$

12. How many nonzero terms of the Maclaurin series for $\ln(1+x)$ do you need to use to estimate $\ln(1.2)$ to within 0.001?

A. 2

B. 3

C. 4

D. 5

E. 6