NAME ______________________________

STUDENT ID ______________________________

REC. INSTR. ________________ REC. TIME ________________

INSTRUCTIONS:

1. Verify that you have all the pages (there are 10 pages).
2. Fill in your name, your student ID number, and your recitation instructor's name and recitation time above. Write your name, your student ID number and division and section number of your recitation section on your answer sheet, and fill in the corresponding circles.
3. Mark the letter of your response for each question on the mark–sense answer sheet.
4. There are 25 problems. Each problem is worth 6 points.
5. No books or notes or calculators may be used.

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!},\ |x| < \infty \qquad \sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1},\ |x| < \infty$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n},\ |x| < \infty \qquad (1+x)^s = \sum_{n=0}^{\infty} \binom{s}{n} x^n,\ |x| < 1$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n},\ |x| < 1 \qquad \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n,\ |x| < 1$$

$$\sin^2 x = \frac{1-\cos(2x)}{2} \qquad \cos^2 x = \frac{1+\cos(2x)}{2}$$

$$\sin(2x) = 2\sin x \cos x \qquad 1 + \tan^2 x = \sec^2 x$$

The angle of rotation θ, $0 < \theta < \pi/2$, that eliminates the xy term from the second degree equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ satisfies the equation $\tan 2\theta = \dfrac{B}{A-C}$, provided $A \neq C$. If $A = C$, then $\theta = \pi/4$.

$x = (\cos\theta)X - (\sin\theta)Y$ and $y = (\sin\theta)X + (\cos\theta)Y$, where the XY coordinate system is obtained by rotation the x and y axes through the angle θ about the origin.

Let R be the region between the graphs of f and g on $[a,b]$. Then the moments of R about x and y axes are

$$M_{y=0} = M_x = \int_a^b \frac{1}{2}(f(x)^2 - g(x)^2)dx$$

$$M_{x=0} = M_y = \int_a^b x(f(x) - g(x))dx.$$

1. Let $\mathbf{u} = \mathbf{i} - \mathbf{j} + t\mathbf{k}$ and $\mathbf{v} = \mathbf{i} + \mathbf{j} - \mathbf{k}$. Find all values t such that $\mathbf{u} \times \mathbf{v}$ has length $2\sqrt{2}$.

A. $t = -1, 2$

B. $t = -3, 2$

C. $t = -2, 3$

D. $t = 0, -1$

E. $t = -1, 1$

2. $\lim_{x \to 0} (1 + 3x)^{\frac{2}{x}} =$

A. 1

B. e^2

C. e^3

D. e^6

E. $e^{\frac{2}{3}}$

3. $\int_1^2 9x^2 \ln x \, dx =$

A. $36 \ln 2$

B. $24 \ln 2 - 7$

C. $24 \ln 2 - 9$

D. $8 \ln 2 - \frac{8}{3}$

E. $8 \ln 2 - \frac{7}{3}$

4. $\displaystyle\int \frac{x-1}{x^2-x-2}\,dx =$

A. $\frac{2}{3}\ln|x+1|+\frac{1}{3}\ln|x-2|+C$

B. $-\frac{2}{3}\ln|x+1|+\frac{1}{3}\ln|x-2|+C$

C. $\frac{1}{3}\ln|x+1|+\frac{2}{3}|x-2|+C$

D. $\frac{1}{3}\ln|x+1|-\frac{2}{3}\ln|x-2|+C$

E. $\frac{2}{3}\ln|x+1|-\frac{2}{3}\ln|x-2|+C$

5. $\displaystyle\int_e^{\infty} \frac{\pi}{x}\,dx =$

A. $-\pi$

B. π^2

C. 0

D. $\frac{\pi}{e}$

E. does not exist

6. The region between $y = x^2$ and $y = x^5$ is rotated about the x-axis. The volume of the resulting figure is

A. $\frac{6}{55}\pi$

B. $\frac{16}{55}\pi$

C. $\frac{1}{3}\pi$

D. $\frac{2}{15}\pi$

E. $\frac{1}{2}\pi$

7. If the curve $y = x^2$, $1 \le x \le 2$, is revolved about the x-axis, then the area of the surface obtained is

A. $\int_1^2 \pi x^4 \sqrt{1 + x^4}\, dx$

B. $\int_1^2 2\pi x^2 \sqrt{1 + x^4}\, dx$

C. $\int_1^2 2\pi x^2 \sqrt{1 + 4x^2}\, dx$

D. $\int_1^2 2\pi x \sqrt{1 + 4x^4}\, dx$

E. $\int_1^2 2\pi x \sqrt{1 + 4x^2}\, dx$

8. A rectangular tank with a square base has dimensions 2 ft $\times$ 2 ft $\times$ 3 ft and is filled with water. How much work is required to pump out all of the water to the top of the tank? (Water has density 62.5 lb/ft^3).

A. 12(62.5) ft-lbs.

B. 18(62.5) ft-lbs.

C. 24(62.5) ft-lbs.

D. 30(62.5) ft-lbs.

E. 36(62.5) ft-lbs.

9. If a triangle has vertices at $(0,0)$, $(2,0)$, $(0,3)$ its center of gravity is at

A. $\left(1, \frac{3}{2}\right)$

B. $\left(\frac{2}{3}, 1\right)$

C. $\left(1, \frac{3}{3}\right)$

D. $\left(\frac{3}{4}, 1\right)$

E. $\left(1, \frac{4}{3}\right)$

10. $\lim\limits_{n\to\infty} \dfrac{n+3}{\sqrt{2n^2+1}} =$

A. 0

B. 1

C. $\dfrac{1}{2}$

D. $\dfrac{1}{\sqrt{2}}$

E. ∞

11. Which of the following statements is true for $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)}$?

A. It converges because $\lim\limits_{n\to\infty} \dfrac{1}{n(\ln n)} = 0$.

B. It converges by ratio test.

C. It diverges by ratio test.

D. It converges by integral test.

E. It diverges by integral test.

12. $\sum_{n=1}^{\infty} 4\left(-\frac{2}{3}\right)^n =$

A. $\frac{12}{5}$

B. $\frac{8}{5}$

C. $-\frac{8}{5}$

D. $-\frac{12}{5}$

E. 8

13. $\int_0^x \sin(t^2)dt =$

A. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+3}}{(2n+1)!(4n+3)}$

B. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+2}}{(2n+2)!}$

C. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+2}}{(2n+1)!}$

D. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+2}}{(2n+1)!(4n+2)}$

E. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+1}}{(2n+1)!(4n+1)}$

14. The series $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n^p+1}}$ converges only if

A. $p > -1$

B. $p > -2$

C. $p > 0$

D. $p > 1$

E. $p > 2$

15. The interval of convergence of $\sum_{n=2}^{\infty} \frac{(n-2)!}{n!} \left(\frac{x}{2}\right)^n$ is

A. $[-1, 1]$

B. $[-1, 1)$

C. $[-2, 2)$

D. $[-2, 2]$

E. $(-\infty, \infty)$

16. Find the first three terms of Taylor series of $x \ln(1 + x^2)$ at 0.

A. $x^2 - \frac{x^3}{2} + \frac{x^4}{3}$

B. $x^2 - \frac{x^4}{2} + \frac{x^6}{3}$

C. $x^3 - \frac{x^5}{2} + \frac{x^7}{3}$

D. $x^2 - \frac{x^4}{2} + \frac{x^6}{6}$

E. $x^3 - \frac{x^5}{2} + \frac{x^7}{6}$

17. Let P be the point with Cartesian coordinates $(\sqrt{3}, -1)$. A set of polar coordinates for P is given by $(r, \theta) =$

A. $\left(1, \frac{\pi}{6}\right)$

B. $\left(2, \frac{\pi}{6}\right)$

C. $\left(-1, \frac{\pi}{6}\right)$

D. $\left(-2, \frac{\pi}{6}\right)$

E. $\left(-2, \frac{5\pi}{6}\right)$

18. The graph of $r = 1 - \sin\theta$ is

A. an ellipse

B. a line

C. a rose

D. a cardioid

E. a circle

19. The circle $(x+3)^2 + y^2 = 9$ has a polar equation given by

A. $r = 6\sin\theta$

B. $r = -6\sin\theta$

C. $r = 6\cos\theta$

D. $r = -6\cos\theta$

E. $r = 3\cos\theta$

20. The curve with parametric equations $x = e^t \cos t$ and $y = e^t \sin t$ $0 \leq t \leq \pi$ has arc length

A. $\sqrt{2}(e^\pi - 1)$

B. $e^\pi - 1$

C. $\sqrt{2}\, e^\pi$

D. e^π

E. e

21. The area of the region inside $r = 2\cos\theta$ and outside $r = \sqrt{3}$ is given by

A. $\frac{1}{2}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (4\cos^2\theta - 3)d\theta$

B. $\frac{1}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} (4\cos^2\theta - 3)d\theta$

C. $\frac{1}{2}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (2\cos\theta - \sqrt{3})^2 d\theta$

D. $\frac{1}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} (2\cos\theta - \sqrt{3})^2 d\theta$

E. $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (4\cos^2\theta - 3)d\theta$

22. The hyperbola $\frac{(x+1)^2}{9} - \frac{(y-1)^2}{16} = 1$ has a focus at

A. $(-1, -3)$

B. $(-1, 6)$

C. $(2, 1)$

D. $(-1, 2)$

E. $(-6, 1)$

23. An ellipse has vertices $(-2, 2)$ and $(-2, -4)$ and eccentricity $\dfrac{2}{3}$. The distance between its foci is

A. 1

B. 2

C. 3

D. 4

E. $2 - \sqrt{6}$

24. The rotation angle θ that eliminates the xy term of $6x^2 + 3xy + 2y^2 + 8 + 5y + 102 = 0$ is

A. $\theta = \cos^{-1}\left(\frac{4}{5}\right)$

B. $\theta = \cos^{-1}\left(\frac{3}{5}\right)$

C. $\theta = \tan^{-1}\left(\frac{3}{4}\right)$

D. $\theta = \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$

E. $\theta = \frac{1}{2}\cos^{-1}\left(\frac{4}{5}\right)$

25. Find an equation of the hyperbola with vertices $(0, -3)$ and $(0, 3)$ and asymptotes $y = \pm\frac{1}{2}\,x$.

A. $-\dfrac{x^2}{4} + \dfrac{y^2}{2} = 1$

B. $-\frac{x^2}{4} + \frac{y^2}{1} = 1$

C. $-\dfrac{x^2}{36} + \dfrac{y^2}{9} = 1$

D. $-\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$

E. $-\dfrac{x^2}{6} + \dfrac{y^2}{3} = 1$