NAME _____

STUDENT ID _____

REC. INSTR. _____ REC. TIME _____

SECTION NUMBER _____ LECTURER _____

INSTRUCTIONS:

1. Make sure you have all 12 test pages.
2. Fill in the information requested above and on the mark-sense sheet.
3. Mark your answers on the mark-sense sheet and show work in this booklet.
4. There are 22 problems, worth 9 points each.
5. No books or notes or calculators may be used.
6. **Please, show your work.** It may matter in borderline cases.
7. Have a good summer.

Formulae you may or may not find useful:

$$\mathrm{pr}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a}\cdot\mathbf{b}}{||\mathbf{a}||^2}\mathbf{a}$$

$$\int \sin^n x\, dx = -\frac{1}{n}\sin^{n-1} x\, \cos\, x + \frac{n-1}{n}\int \sin^{n-2} x\, dx$$

$$\int \cos^n x\, dx = \frac{1}{n}\cos^{n-1} x \sin\, x + \frac{n-1}{n}\int \cos^{n-2} x\, dx$$

$$\int \sec x\, dx = \ln|\sec\, x + \tan\, x| + C$$

$$\ln(1+x) = \sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}$$

$$\tan^{-1} x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1}$$

$$(1+x)^s = \sum_{n=0}^{\infty}\binom{s}{n}x^n$$

$$\tan 2\theta = \frac{B}{A-C}$$

$$x = X\cos\theta - Y\sin\theta, \quad y = X\sin\theta + Y\cos\theta.$$

1. The vector $\mathbf{v} = v_1\,\mathbf{i} + v_2\,\mathbf{j} + v_3\,\mathbf{k}$ has length 3 and the same direction as $4\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$. Then $v_1 =$

   A. 3
   B. 2
   C. 1
   D. 0
   E. -2

2. You have to push your broken car 50 meters, by exerting a force of 200 Newtons, at angle $\pi/6$ with respect to the road. How much work will you do on the car?

   A. $10,000\sqrt{3}$ Nm
   B. $10,000$ Nm
   C. $5,000\sqrt{3}$ Nm
   D. $5,000$ Nm
   E. none of the above

3. A vector perpendicular to both $\mathbf{i}+2\mathbf{j}$ and $\mathbf{j}+2\mathbf{k}$ is

A. $2\mathbf{i}-\mathbf{j}+4\mathbf{k}$
B. $\mathbf{i}+4\mathbf{j}+2\mathbf{k}$
C. $4\mathbf{i}+\mathbf{j}-2\mathbf{k}$
D. $2\mathbf{i}-4\mathbf{j}-\mathbf{k}$
E. $4\mathbf{i}-2\mathbf{j}+\mathbf{k}$

4. $\lim_{x\to 1^+} \ln x \ln(\ln x) =$

A. 0
B. 1
C. e
D. ∞
E. $-\infty$

5. $\int_0^1 (x-1)\, e^{x/2}\, dx =$

A. $2\sqrt{e}$
B. 0
C. $6 - 4\sqrt{e}$
D. $\sqrt{e} - 2$
E. 1

6. In computing $\int \sin^{-2} x \cos^3 x dx$ which of the following steps will be used?

A. integrate by parts
B. do partial fractions
C. substitute $u = \sin x$
D. substitute $u = \cos x$
E. substitute $u = \sec x$

7. The partial fraction expansion of the function $\dfrac{x^3+2}{x^2-1}$ will be of form

A. $\dfrac{A}{x^2-1}+\dfrac{Bx+C}{x^3+1}$

B. $x+\dfrac{A}{x-1}+\dfrac{B}{x+1}$

C. $x^3+\dfrac{A}{x-1}+\dfrac{B}{x+1}$

D. $\dfrac{A}{x-1}+\dfrac{B}{x+1}$

E. $\dfrac{3x}{2}+\dfrac{Ax+B}{x^2-1}$

8. $\displaystyle\int_0^2 \frac{dx}{(4+x^2)^{3/2}} =$

A. $\dfrac{\sqrt{2}}{2}$

B. $\dfrac{\sqrt{2}}{8}$

C. 1

D. $\dfrac{\pi}{4}$

E. $\dfrac{\pi}{8}$

9. The base of a solid is an isosceles right triangle, with legs of length 3. The cross sections perpendicular to one leg are squares. What is the volume of the solid?

A. 6
B. $9\sqrt{2}$
C. 9
D. $\frac{27}{2}$
E. $\frac{27}{\sqrt{2}}$

10. Two kids are sitting on opposite sides of a seesaw, both 1 ft from the axis of revolution. One kid weighs 10 lbs, the other 100 lbs. How far from the axis should a third kid, also weighing 10 lbs, sit to achieve equilibrium?

A. 6 ft
B. 8 ft
C. 9 ft
D. 10 ft
E. 11 ft

11. $\lim_{k\to\infty} \frac{2\ln k}{\sqrt{k+1}} =$

A. 2
B. $\frac{1}{2}$
C. $\frac{1}{4}$
D. 0
E. 1

12. $\sum_{n=0}^{\infty} \frac{(-1)^n}{2} 3^{1-n} =$

A. $\frac{9}{8}$
B. $\frac{3}{4}$
C. $\frac{9}{4}$
D. $\frac{3}{2}$
E. 3

13. Which of the following statements is/are true?

I. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ converges;

II. $\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$ converges by the limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{2^n}$;

III. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2 + 1}$ converges absolutely.

A. Only I
B. Only II and III
C. Only III
D. Only II
E. All three are true.

14. For what positive values of d does $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^d + 1}}$ converge?

A. $0 < d \leq 1$
B. $1 < d < \infty$
C. $2 < d < \infty$
D. $\frac{1}{2} < d < \infty$
E. $0 < d < \infty$

15. If $\frac{d}{dx}(\frac{\sin x}{x})$ is written as $\sum_{n=0}^{\infty} a_n x^n$ then a_5 equals

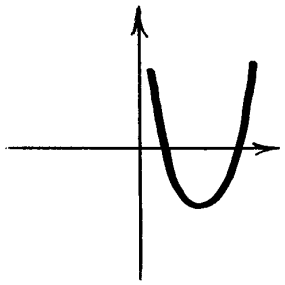
A. $-\frac{1}{6!}$
B. $-\frac{6}{7!}$
C. $-\frac{1}{7!}$
D. $\frac{1}{6 \cdot 5!}$
E. $\frac{5}{6!}$

16. The Taylor series of the function $\frac{x^2}{1+2x^3}$ is

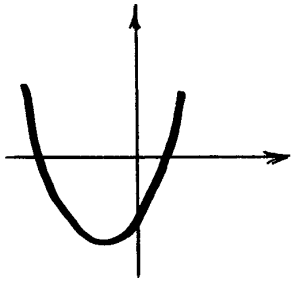
A. $1 - x^2 + 2x^3 + \ldots$
B. $x^2 - 2x^3 + 2x^4 + \ldots$
C. $x - 2x^3 + 4x^5 + \ldots$
D. $x^2 - 2x^5 + 4x^8 + \ldots$
E. $2x^3 - 4x^5 + 8x^7 + \ldots$

17. Which of the following curves is parametrized by $x = 2 - t^2$, $y = t - 1$?

A.



B.



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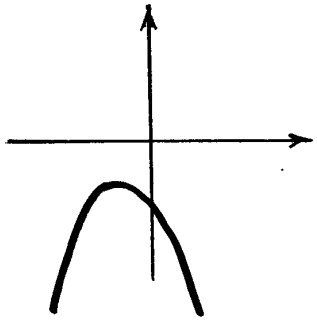

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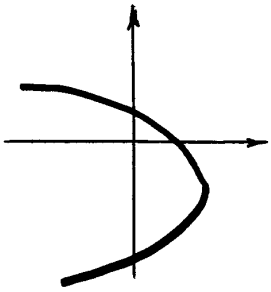
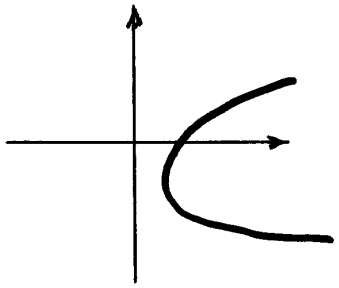


E.



18. If a particle travels along the path $x = t^2 - 1$, $y = 2t^3 - 5t^2$, what is its velocity at time $t = 2$?

A. $5\sqrt{3}$
B. 8
C. $4\sqrt{2}$
D. 4
E. $2\sqrt{6}$

19. Find the area of the region surrounded by the curve $r = \sqrt{1 + \cos\theta}$.

A. $\frac{\pi}{2}$
B. π
C. 4π
D. $\frac{\pi}{2} + 1$
E. $\pi + 2$

20. The equation $r = 2\cos\theta - 4\sin\theta$ describes a circle. What is its center?

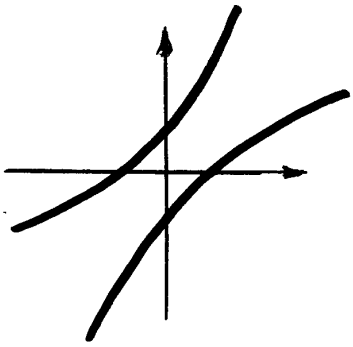
A. $(-1, 2)$
B. $(\frac{1}{2}, -1)$
C. $(2, 4)$
D. $(2, -4)$
E. $(1, -2)$

21. Find the foci of the ellipse $x^2 - 4x + 4y^2 - 8y + 4 = 0$.

A. $(2, 1 \pm \sqrt{5})$
B. $(-2, \pm\sqrt{3}, -1)$
C. $(\pm\sqrt{3}, 0)$
D. $(0, \pm\sqrt{3})$
E. $(2 \pm \sqrt{3}, 1)$

22. Which conic section is described by the equation $x^2 - 4xy + y^2 = 1$

A.



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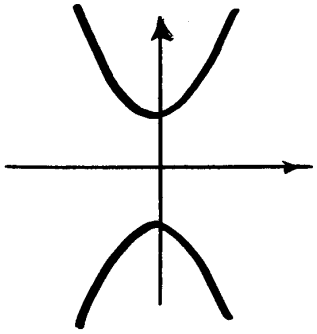

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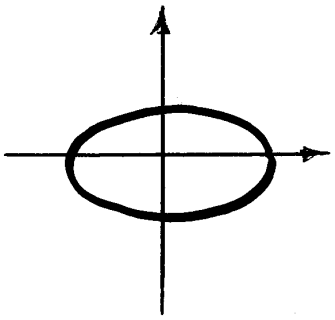
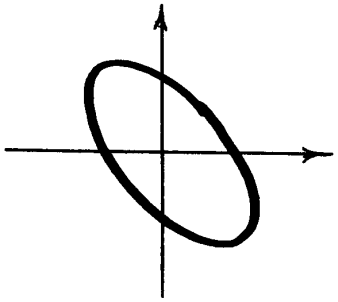


D.



E.