From snider@math.purdue.edu Tue Jun 19 09:48:07 2001 Date: Tue, 19 Jun 2001 09:24:16 -0500 (E
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NAME ______________________________

STUDENT ID ______________________________

REC. INSTR. ________________ REC. TIME ________________

INSTRUCTIONS:

1. Verify that you have all the pages (there are 10 pages).
2. Fill in your name, your student ID number, and your recitation instructor's name and recitation time above. Write your name, your student ID number and division and section number of your recitation section on your answer sheet, and fill in the corresponding circles.
3. Mark the letter of your response for each question on the mark–sense answer sheet.
4. There are 25 problems. Each problem is worth 6 points.
5. No books or notes or calculators may be used.
6. Show your work in the booklet. This may help you if you fall near a letter grade borderline.

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!},\ |x| < \infty$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1},\ |x| < \infty$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n},\ |x| < \infty$$

$$(1+x)^s = \sum_{n=0}^{\infty} \binom{s}{n} x^n,\ |x| < 1$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n},\ |x| < 1$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n,\ |x| < 1$$

$$\sin^2 x = \frac{1-\cos(2x)}{2}$$

$$\cos^2 x = \frac{1+\cos(2x)}{2}$$

$$\sin(2x) = 2\sin x \cos x$$

$$1 + \tan^2 x = \sec^2 x$$

1. Find the radius of the sphere whose equation is

$$x^2 + y^2 + z^2 - 4x + 2y + 4z + 5 = 0$$

A. 1

B. 2

C. 3

D. 4

E. 5

2. Find the area of the parallelogram which is spanned by $\mathbf{u} = \mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{v} = -2\mathbf{i} - \mathbf{j} - 3\mathbf{k}$.

A. $\sqrt{42}$

B. $5\sqrt{2}$

C. $\sqrt{43}$

D. $\sqrt{26}$

E. $\sqrt{14}$

3. Find the area of the region in the first quadrant bounded by the curves $y = 2x^2$ and $y = 12 - x^2$.

A. $\frac{64}{3}$

B. $\frac{80}{3}$

C. 8

D. 16

E. 32

4. The region bounded by the curves $y = x^2$, $y = 0$, $x = 1$, and $x = 2$ is rotated about the y-axis. Find the volume of the solid obtained.

A. $\frac{3}{2}\pi$

B. 2π

C. $\frac{9}{2}\pi$

D. $\frac{31}{5}\pi$

E. $\frac{15}{2}\pi$

5. $\int_0^{\frac{\pi}{4}} x\cos(2x)dx =$

A. $\frac{\pi - 2}{8}$

B. $\frac{\pi + 2}{8}$

C. $\frac{\pi^2}{64}$

D. $\frac{\pi^2 - 16}{64}$

E. $\frac{1}{4}$

6. $\int_0^{\frac{\pi}{2}} \sin^3 x \cos^2 x \, dx =$

A. $\frac{1}{3}$

B. $\frac{1}{4}$

C. $\frac{3}{5}$

D. $\frac{1}{12}$

E. $\frac{2}{15}$

7. $\displaystyle\int_{-1}^{0} \frac{x+5}{x^2+x-2}\,dx =$

A. $-\ln 2$

B. $\ln 2$

C. $-3\ln 2$

D. $\ln 2 - \ln 3$

E. $2\ln 3$

8. $\displaystyle\int_{0}^{\infty} \frac{x}{(x^2+2)^3}\,dx =$

A. $\frac{1}{4}$

B. $\frac{1}{8}$

C. $\frac{1}{16}$

D. $\frac{1}{32}$

E. the integral diverges

9. $\displaystyle\lim_{n\to\infty} \frac{(n^2+1)2^{n+1}}{(n^2-n)3^{n-1}} =$

A. ∞

B. 1

C. 6

D. 0

E. $\frac{2}{3}$

10. Which of the following series converge?

(I) $\sum_{n=1}^{\infty} \frac{n^3}{2^n}$

(II) $\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n}$

(III) $\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right)$

A. (I) and (II) only

B. (I) and (III) only

C. (II) and (III) only

D. (I) only

E. (II) only

11. Which of the following series converge absolutely?

(I) $\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$

(II) $\sum_{n=1}^{\infty} (-1)^n \sin\left(\frac{1}{n}\right)$

(III) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt[3]{n^2+1}}$

A. (I) and (II) only

B. (I) and (III) only

C. (II) and (III) only

D. (I) only

E. (III) only

12. The series $\sum_{n=1}^{\infty} 5 \frac{2^{n+1}}{7^n} =$

A. diverges

B. 14

C. $\frac{10}{7}$

D. 7

E. 4

13. The radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{2^n x^n}{n!}$ is

A. $\frac{1}{2}$

B. ∞

C. 1

D. 2

E. 0

14. Use the Taylor series of $\sin(x^2)$ to approximate $\int_0^1 \sin(x^2)dx$ with error less than 0.001. The smallest number of nonzero terms of the series that are needed for this accuracy is

A. 1

B. 2

C. 3

D. 4

E. 5

15. The Maclaurin series for $e^{\frac{x^2}{2}+1}$ is given by

A. $\sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$

B. $\sum_{n=0}^{\infty} \frac{x^n}{2^n n!}$

C. $\sum_{n=0}^{\infty} \frac{ex^n}{2^n n!}$

D. $\sum_{n=0}^{\infty} \frac{e^n x^{2n}}{2^n n!}$

E. $\sum_{n=0}^{\infty} \frac{ex^{2n}}{2^n n!}$

16. The length of the graph of $f(x) = 1 - \frac{2}{3}x^{\frac{3}{2}}$, $0 \le x \le 1$ is equal to

A. $\frac{2}{3}$

B. $\frac{2}{3}\,(2^{\frac{3}{2}})$

C. $\frac{2}{3}\,(2^{\frac{3}{2}} - 1)$

D. $\frac{1}{3}$

E. $\frac{1}{3}\,(2^{\frac{3}{2}})$

17. Find a Cartesian equation for the parametric curve $x = \frac{t-1}{2}$, $y = t^2 + 1$.

A. $y = x^2 + 1$

B. $y = \frac{x^2 + 2x + 3}{4}$

C. $y = \frac{x^2 - 2x + 3}{4}$

D. $y = 4x^2 + 4x + 2$

E. $y = 4x^2 - 4x + 2$

18. Find the slope of the tangent line to the parametric curve at the point where $t = 1$; $x = t^2 + 2t$, $y = t^3$.

A. $\frac{1}{3}$

B. $\frac{3}{4}$

C. $\frac{4}{3}$

D. 3

E. 4

19. Find an integral for the area of the surface obtained by rotating the curve about the x-axis: $x = 2\cos t,\ y = 3\sin t,\ 0 \le t \le \dfrac{\pi}{3}$

A. $\displaystyle\int_0^{\frac{\pi}{3}} 2\pi\sqrt{1 + 9\cos^2 t}\,dt$

B. $\displaystyle\int_0^{\frac{\pi}{3}} 4\pi\cos t\sqrt{1 + 9\cos^2 t}\,dt$

C. $\displaystyle\int_0^{\frac{\pi}{3}} 6\pi\sin t\sqrt{1 + 9\cos^2 t}\,dt$

D. $\displaystyle\int_0^{\frac{\pi}{3}} 4\pi\cos t\sqrt{4\sin^2 t + 9\cos^2 t}\,dt$

E. $\displaystyle\int_0^{\frac{\pi}{3}} 6\pi\sin t\sqrt{4\sin^2 t + 9\cos^2 t}\,dt$

20. Find the Cartesian coordinates of the center of the circle $r = -2\cos\theta$.

A. $(0, 0)$

B. $(1, 0)$

C. $(-1, 0)$

D. $(0, 1)$

E. $(0, -1)$

21. Which picture most resembles the graph of the curve $r = -\sin 3\theta$?

22. The area of the region that lies inside $r = 1+\cos\theta$ and outside $r = 3\cos\theta$ is represented by the integral

A. $\int_0^{\frac{\pi}{3}} [(1+\cos\theta)^2 - (3\cos\theta)^2]d\theta$

B. $\int_0^{\frac{\pi}{3}} [(3\cos\theta)^2 - (1+\cos\theta)^2]d\theta$

C. $\int_{\frac{\pi}{3}}^{\pi} [(1+\cos\theta)^2 - (3\cos\theta)^2]d\theta$

D. $\int_{\frac{\pi}{3}}^{\pi} \frac{1}{2}[(1+\cos\theta)^2 - (3\cos\theta)^2]d\theta$

E. $\int_{\frac{\pi}{3}}^{\pi} [(3\cos\theta)^2 - (1+\cos\theta)^2]d\theta$

23. Find the foci of the conic section given by the equation $4x^2 - 8x + y^2 + 4y + 4 = 0$.

A. $(1, -3)$ and $(1, -1)$

B. $(1 \pm \sqrt{3}, -2)$

C. $(1, 0)$ and $(1, -4)$

D. $(3, -2)$ and $(-1, -2)$

E. $(1, -2 \pm \sqrt{3})$

24. A conic section is given by the polar equation $r = \dfrac{5}{2 - 2\sin\theta}$. An equation of the directrix of this conic section is given by

A. $x = -\frac{5}{2}$

B. $y = -5$

C. $y = \frac{5}{2}$

D. $y = -\frac{5}{2}$

E. $y = -5$

25. An equation of a parabola with focus $(1, -1)$ and directrix $y = 5$ is given by

A. $y^2 - 4y + 12x - 8 = 0$

B. $y^2 - 2y + 12x - 23 = 0$

C. $x^2 - 4x + 12y - 8 = 0$

D. $x^2 - 2x + 12y - 23 = 0$

E. $y^2 - 2y + 6x - 11 = 0$