MA 162 Exam 1 Spring 2000

NAME SOLUTION KEY

STUDENT ID

REC. INSTR. REC. TIME

INSTRUCTIONS:

1. Verify that you have all the pages (there are 7 pages).
2. Fill in your name, your student ID number, and your recitation instructor's name and recitation time above. Write your name, your student ID number and division and section number of your recitation section on your answer sheet, and fill in the corresponding circles.
3. Mark the letter of your response for each question on the mark–sense answer sheet.
4. There are 11 problems worth 9 points each.
5. No books or notes or calculators may be used.
6. Please hand in both your answer sheet and exam to your recitation instructor,

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$$\int \sec x \, dx = \ln|\sec x + \tan x| + C$$

$$\sin^2 x = \frac{1 - \cos(2x)}{2}$$

$$\cos^2 x = \frac{1 + \cos(2x)}{2}$$

$$\sin(2x) = 2 \sin x \cos x$$

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1. Suppose that $\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$ is a vector that is perpendicular to both $\vec{v} = \vec{i} - \vec{j}$ and $\vec{w} = \vec{i} - 2\vec{j} + 2\vec{k}$. If $a_3 = 1$, then $a_2$ equals

$\vec{a}$ perpendicular to $\pm(v \times w)$

$$v \times w = \begin{vmatrix} i & j & k \\ 1 & -1 & 0 \\ 1 & -2 & 2 \end{vmatrix}$$

$$= i(-2-0) - j(2-0) + k(-2+1)$$

$$= -2i - 2j - k$$

$$= \pm(a_1 i + a_2 j + a_3 k)$$

$$a_3 = 1 \rightarrow a_2 = 2$$

A. 1
B. $\frac{1}{2}$
C. $-1$
D. 2
E. $-\frac{1}{2}$

2. Find the angle $\theta$ between the vectors $\vec{a} = \vec{i} + \vec{j}$ and $\vec{b} = \vec{i} + \vec{j} + 4\vec{k}$.

$$\cos\theta = \frac{a \cdot b}{\|a\| \, \|b\|}$$

$$= \frac{(1)(1) + (1)(1) + (0)(4)}{\sqrt{1^2+1^2+0^2}\,\sqrt{1^2+1^2+4^2}}$$

$$= \frac{2}{\sqrt{2}\,\sqrt{18}}$$

$$= \frac{1}{3} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{3}\right)$$

A. $\cos^{-1}\left(\frac{2}{3}\right)$
B. $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$
C. $\cos^{-1}\left(\frac{1}{3}\right)$
D. $\cos^{-1}\left(\frac{2}{9}\right)$
E. $\cos^{-1}\left(\frac{1}{\sqrt{6}}\right)$

3. Compute $\lim_{x\to 0^+} (2x^2)^{3x}$. $= 0^0$.

$$\lim_{x\to 0^+} e^{\ln(2x^2)^{3x}} = \lim_{x\to 0^+} e^{\ln(2x)^{6x}}$$

$$= e^{\lim_{x\to 0^+} \ln(2x)^{6x}} \overset{(*)}{=} e^0 = 1.$$

A. $\frac{2}{3}$

B. $e^{\frac{2}{3}}$

C. $\frac{\sqrt{2}}{3}$

D. 1

E. 0

$$(*) \quad \lim_{x\to 0^+} \ln(2x)^{6x} = \lim_{x\to 0^+} (6x)\ln(2x)$$

$$= \lim_{x\to 0^+} \frac{\ln 2x}{\frac{1}{6x}} \overset{\frac{-\infty}{\infty}}{=} \lim_{x\to 0^+} \frac{\frac{2}{2x}}{\frac{1}{6}\left(\frac{-1}{x^2}\right)} = \lim_{x\to 0^+} (-6x) = 0$$

4. Compute $\int_0^1 x4^x\,dx$.

Let $u = x$ and $dv = 4^x dx$

Then $du = dx$ and $v = \frac{1}{\ln 4} 4^x$

A. $\frac{4}{\ln 4} - 1$

B. $\frac{4}{\ln 4} - \frac{3}{(\ln 4)^2}$

C. $\frac{4}{\ln 4} - \frac{1}{(\ln 4)^2}$

D. $4 - \frac{2}{\ln 4}$

E. $2 - \frac{1}{\ln 4} + \frac{1}{(\ln 4)^2}$

$$\int_0^1 x4^x dx = x\frac{1}{\ln 4}4^x \Big|_0^1 - \int_0^1 \frac{1}{\ln 4} 4^x dx$$

$$= \left(x\frac{1}{\ln 4}4^x - \frac{1}{(\ln 4)^2}4^x\right)\Big|_0^1$$

$$= \left(\frac{4}{\ln 4} - \frac{4}{(\ln 4)^2}\right) - \left(0 - \frac{1}{(\ln 4)^2}\right)$$

$$= \frac{4}{\ln 4} - \frac{3}{(\ln 4)^2}$$

5. Compute $\int_0^{\frac{\pi}{8}} \tan 2x \sec^2 2x\, dx$.

A. $\frac{1}{4}$
B. $\frac{1}{8}$
C. 1
D. $\frac{1}{2}$
E. 2

Let $u = \tan 2x$

then $du = (\sec^2 2x)(2)\, dx$

$u(0) = \tan 0 = 0$ , $u\left(\frac{\pi}{8}\right) = \tan \frac{\pi}{4} = 1$

$$\int_0^{\pi/8} \tan 2x \sec^2 2x\, dx$$

$$= \int_0^1 u\left(\tfrac{1}{2} du\right) = \tfrac{1}{4} u^2 \Big|_0^1 = \tfrac{1}{4}(1-0) = \tfrac{1}{4}$$

6. To evaluate $\int \sqrt{x^2+2x}\, dx$ which substitution should be used?

A. $x = \tan t - 1$
B. $x = \sec t - 1$
C. $x = 1 + \sin t$
D. $x = 1 - \sin t$
E. $x = 1 - \sin 2t$

complete square: $x^2+2x+1$

Therefore $x^2+2x = x^2+2x+1-1$

$= (x+1)^2 - 1$

Thus let $x+1 = \sec t$

that is $x = \sec t - 1$

7. Compute $\int_4^6 \frac{2x-4}{x^2-4x+3}\,dx$.

A. $\ln\frac{3}{5}$

B. $\ln 6$

C. $\ln\frac{5}{3}$

D. $\ln 15$

E. $\ln 5$

$$\frac{2x-4}{x^2-4x+3} = \frac{2x-4}{(x-3)(x-1)} = \frac{A}{x-3} + \frac{B}{x-1}$$

$$\rightarrow 2x-4 = A(x-1) + B(x-3)$$

$$x=3 \rightarrow 2 = 2A + 0B \rightarrow A = 1$$

$$x=1 \rightarrow -2 = 0A - 2B \rightarrow B = 1$$

$$\int_4^6 \left(\frac{1}{x-3} + \frac{1}{x-1}\right)dx = \left(\ln|x-3| + \ln|x-1|\right)\Big|_4^6$$

$$= (\ln 3 + \ln 5) - (\ln 1 + \ln 3)$$

$$= \ln 5$$

8. Compute the improper integral $\int_0^\infty \frac{8x\,dx}{(2x^2+1)^3}$.

A. 1

B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. 2

E. integral diverges

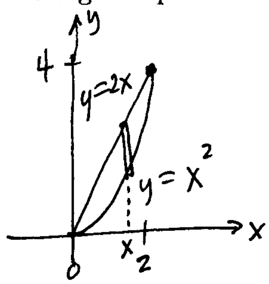
$$= \lim_{c\to\infty} \int_0^c \frac{8x\,dx}{(2x^2+1)^3}$$

$$= \lim_{c\to\infty} (2)(-\tfrac{1}{2})(2x^2+1)^{-2}\Big|_0^c$$

$$= \lim_{c\to\infty} \left(-\frac{1}{(2c^2+1)^2} + 1\right)$$

$$= 1.$$

9. The base of a solid $S$ is the region in the $xy$-plane between the curves $y = 2x$ and $y = x^2$. The cross-section of $S$ perpendicular to the $x$-axis is a semicircle. Which integral represents the volume of $S$?




Diameter of semicircle at x is $2x - x^2$.

Semicircle cross section at x has area $\frac{1}{2}\pi\left(\frac{2x-x^2}{2}\right)^2$

Volume = $\int_0^2 \frac{\pi}{2}\left(\frac{2x-x^2}{2}\right)^2 dx$

A. $\int_0^2 \frac{\pi}{2}(x^4 - 4x^2)^2 dx$

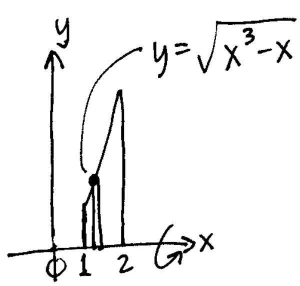
B. $\int_0^2 \frac{\pi}{2}\left(\frac{2x - x^2}{2}\right)^2 dx$

C. $\int_0^2 \pi\left(\frac{x^4}{4} - x^2\right) dx$

D. $\int_0^2 \pi(2x - x^2)^2 dx$

E. $\int_0^2 \frac{\pi}{2}\sqrt{2x - x^2}\, dx$

10. Let $R$ be the region lying under the curve $y = \sqrt{x^3 - x}$ for $1 \le x \le 2$. Compute the volume of the solid obtained by revolving $R$ about the $x$-axis.




$V = \int_1^2 \pi\left(\sqrt{x^3-x}\right)^2 dx$

$= \int^2 \pi(x^3 - x)\, dx = \pi\left(\frac{1}{4}x^4 - \frac{1}{2}x^2\right)\Big|_1^2$

$= \pi(4-2) - \pi\left(\frac{1}{4} - \frac{1}{2}\right)$

$= \frac{9}{4}\pi$

A. $\frac{7\pi}{4}$

B. $\frac{11\pi}{4}$

C. $2\pi$

D. $\frac{3\pi}{2}$

E. $\frac{9\pi}{4}$

11. Two children are pulling on a sled as shown in the diagram. If one child pulls on rope $A$ with a force of 20 lbs, and if the other pulls on rope $B$ with a force of $F$ lbs, what must $F$ be for the sled to move along the line $\ell$?

A. $15\sqrt{2}$

B. $20\sqrt{3}$

C. $10\sqrt{6}$

D. $20\sqrt{2}$

E. $20\sqrt{6}$