MA 162 Exam 2 Spring 2000

NAME SOLUTION KEY

STUDENT ID ____________________

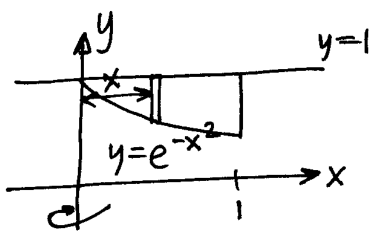
REC. INSTR. ____________________ REC. TIME. ____________________

INSTRUCTOR ____________________

INSTRUCTIONS:

1. Make sure that you have all 7 test pages.
2. Fill in the information requested above and on the answer sheet.
3. Mark the letter of your response for each question on the mark–sense answer sheet.
4. There are 10 problems worth 9 points each, and 2 worth 5 points each for a total of 100 points.
5. No books or notes or calculators may be used.

1. Suppose R is the region between the graphs of $y = e^{(-x^2)}$ and $y = 1$, above the interval $0 \le x \le 1$. Find the volume of the solid obtained by revolving R about the y axis.



Shell Method:

$$V = \int_0^1 2\pi x \left(1 - e^{-x^2}\right) dx$$

$$= 2\pi \int_0^1 \left(x - x e^{-x^2}\right) dx = 2\pi \left(\frac{x^2}{2} + \frac{e^{-x^2}}{2}\right)\Big|_0^1$$

$$= 2\pi\left(\frac{1}{2} + \frac{e^{-1}}{2}\right) - 2\pi\left(0 + \frac{1}{2}\right) = \frac{\pi}{e}$$

A. $2\pi - \frac{\pi}{e}$

B. $\pi + \frac{\pi}{e}$

C. $\pi - \frac{\pi}{e}$

D. $\frac{\pi}{e}$

E. $\pi + \frac{2\pi}{e}$

2. Find the length of the curve $y = 2x^{\frac{3}{2}}$ for $0 \le x \le \frac{1}{3}$.

$$y' = 3x^{1/2}$$

$$L = \int_0^{\frac{1}{3}} \sqrt{1 + \left(3x^{1/2}\right)^2}\, dx$$

$$= \int_0^{1/3} \sqrt{1 + 9x}\, dx$$

$$= \frac{1}{9} \cdot \frac{2}{3} (1 + 9x)^{3/2} \Big|_0^{\frac{1}{3}}$$

$$= \frac{2}{27}\left(4^{3/2} - 1^{3/2}\right) = \frac{2}{27}(7) = \frac{14}{27}$$

A. $\frac{7}{27}$

B. $\frac{7}{9}$

C. $\frac{14}{27}$

D. $\frac{2}{9}$

E. $\frac{16}{27}$

3. Suppose that a force of 2 lbs is needed to stretch a spring $\frac{1}{2}$ ft. beyond its natural length. Calculate the work required to stretch it an additional $\frac{1}{2}$ ft.

Hooke's Law: $F = kx$, $2 = k\frac{1}{2} \to k = 4$

$$\text{Work} = \int_{0.5}^{1} 4x\,dx = 2x^2 \Big|_{0.5}^{1}$$

$$= 2(1 - 0.25) = 2(0.75) = 1.5$$

A. $\frac{3}{2}$ ft-lbs

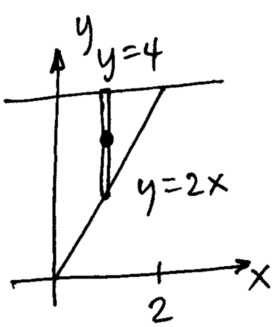
B. 2 ft-lbs

C. $\frac{1}{2}$ ft-lbs

D. 1 ft-lb

E. 3 ft-lbs

4. A plate occupies the part of the first quadrant between the lines $y = 2x$ and $y = 4$. Find the x coordinate of center of gravity.



$$\bar{x} = \frac{M_{x=0}}{\text{mass}} = \frac{M_y}{\text{mass}} = \frac{\frac{8}{3}}{4} = \frac{2}{3}$$

(see below)

A. 1

B. $\frac{2}{3}$

C. $\frac{4}{3}$

D. $\frac{3}{2}$

E. $\frac{1}{2}$

$$\text{mass} = \int_0^2 \rho(4-2x)\,dx = \rho(4x - x^2)\Big|_0^2 = \rho(4)$$

$$M_{x=0} = \int_0^2 x\rho(4-2x)\,dx = \int_0^2 \rho(4x - 2x^2)\,dx$$

$$= \rho\left(2x^2 - \frac{2}{3}x^3\right)\Big|_0^2 = \rho\left(8 - \frac{2}{3}(8)\right) = \rho\left(\frac{8}{3}\right)$$

5. If the third Taylor polynomial of a given function $f(x)$ is $p_3(x) = 1 - x + \frac{x^2}{3} + \frac{x^3}{2}$ then the values of $f''(0)$ and $f'''(0)$ are, respectively

A. $\frac{2}{3}$ and 3

B. $\frac{1}{3}$ and $\frac{1}{2}$

C. $\frac{2}{3}$ and $\frac{1}{2}$

D. $\frac{1}{6}$ and 3

E. both are 3

Recall: $p_3(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3$

Therefore: $\frac{f''(0)}{2!}x^2 = \frac{1}{3}x^2 \rightarrow \frac{f''(0)}{2} = \frac{1}{3}$

$\rightarrow f''(0) = \frac{2}{3}$

and $\frac{f'''(0)}{3!}x^3 = \frac{1}{2}x^3 \rightarrow \frac{f'''(0)}{3!} = \frac{1}{2}$

$\rightarrow f'''(0) = \frac{3 \cdot 2}{2} = 3$

6. $\lim_{m\to\infty} \frac{\sqrt{m^2+m-1}}{2m-1} =$

A. ∞

B. 2

C. 1

D. $\frac{1}{2}$

E. 0

$$= \lim_{m\to\infty} \left(\frac{\sqrt{m^2+m-1}}{2m-1}\right)\left(\frac{\frac{1}{m}}{\frac{1}{m}}\right)$$

$$= \lim_{m\to\infty} \left(\frac{\sqrt{\frac{m^2+m-1}{m^2}}}{\frac{2m-1}{m}}\right)$$

$$= \lim_{m\to\infty} \frac{\sqrt{1+\frac{1}{m}-\frac{1}{m^2}}}{2-\frac{1}{m}} = \frac{\sqrt{1}}{2} = \frac{1}{2}$$

7. $\lim_{k\to\infty} (\sqrt{k+5} - \sqrt{k}) =$ $\infty - \infty$

$$= \lim_{k\to\infty} \left(\sqrt{k+5} - \sqrt{k}\right)\left(\frac{\sqrt{k+5} + \sqrt{k}}{\sqrt{k+5} + \sqrt{k}}\right)$$

$$= \lim_{k\to\infty} \frac{k+5-k}{\sqrt{k+5}+\sqrt{k}} = \lim_{k\to\infty} \frac{5}{\sqrt{k+5}+\sqrt{k}} = 0$$

A. $-\frac{1}{2}$
B. 0
C. 1
D. $\sqrt{5}$
E. ∞

8. $\sum_{n=0}^{\infty} \frac{3^{n-2}}{4^{n+1}} =$ $\sum_{n=0}^{\infty} \frac{3^{-2} \cdot 3^n}{4 \cdot 4^n}$

$$= \sum_{n=0}^{\infty} \frac{1}{36}\left(\frac{3}{4}\right)^n$$

$$= \frac{1}{36}\left(\frac{1}{1-\frac{3}{4}}\right)$$

$$= \frac{1}{36}(4) = \frac{1}{9}$$

A. $\frac{1}{4}$
B. $\frac{1}{9}$
C. $\frac{3}{16}$
D. 1
E. the series diverges

9. The series $\sum_{n=1}^{\infty} \frac{1}{2n - \sqrt{n}}$

A. converges by the integral test

B. converges by the limit comparison test

C. converges by the alternating series test

D. diverges by the ratio test

E. diverges by the limit comparison test

$\sum^{\infty} \frac{1}{n}$ diverges

and $\lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{2n-\sqrt{n}}}$

$= \lim_{n\to\infty} \frac{2n-\sqrt{n}}{n} = 2$

Therefore $\sum^{\infty} \frac{1}{n}$ and $\sum^{\infty} \frac{1}{2n-\sqrt{n}}$ both diverge by limit comparison test.

10. Which of the following series converges?

I. $\sum_{n=1}^{\infty} (-1)^n \frac{n+1}{n}$; diverges

II. $\sum_{n=0}^{\infty} \frac{2^n}{n!}$; converges

III. $\sum_{n=0}^{\infty} e^n$ diverges.

A. only I

B. only II

C. only III

D. only I and II

E. none of the series converge

I. and III. $\lim_{n\to\infty} a_n \neq 0$.

II. Converges by Ratio Test

$a_{n+1} \cdot \frac{1}{a_n} = \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n} = 2 \cdot \frac{1}{n+1} \Rightarrow 0$ as $n \to \infty$

11. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1}$

A. diverges

B. converges absolutely

C. converges conditionally

$\sum^{\infty} \frac{(-1)^n}{2n+1}$ converges by Alternating Series Test. ($a_n \to 0$ and $|a_n|$ are decreasing.)

However $\sum^{\infty} \frac{1}{2n+1}$ diverges (use limit comparison with divergent $\sum^{\infty} \frac{1}{n}$)

Therefore $\sum^{\infty} \frac{(-1)^n}{2n+1}$ converges conditionally

12. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\sqrt{n}}$

A. diverges

B. converges absolutely

C. converges conditionally

converges absolutely since

$\sum^{\infty} \frac{1}{n\sqrt{n}} = \sum^{\infty} \frac{1}{n^{3/2}}$ is a convergent p-series.